NLO BFKL and anomalous dimensions of light-ray operators

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Outline

- Light-cone limit of Cornalba's formula.
- Light-ray operators.
- DGAP vs BFKL for 4-point correlators.

$$x_{12}^4 x_{34}^4 \langle \mathcal{O}(x_1)\mathcal{O}^\dagger(x_2)\mathcal{O}(x_3)\mathcal{O}^\dagger(x_4)\rangle \;=\; \frac{i}{2}\int d\nu\, \frac{\tanh\pi\nu}{\nu} F(\nu)\Omega(r,\nu) R^{\frac{1}{2}\aleph(\nu)}\tilde{f}_+(\aleph(\nu))$$

$\aleph(\nu)$ - pomeron intercept, $\tilde{f}_+(\omega) = (e^{i\pi\omega} - 1)/\sin\pi\omega$ - signature factor

$$\Omega(r,\nu) \;=\; \frac{\nu}{2\pi^2}\frac{\sin 2\nu\rho}{\sinh\rho}, \quad \cosh\rho = \frac{\sqrt{r}}{2}$$

In the limit Regge + $x_{12}^2 \to 0$

$$R = \frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} \;\to\; \frac{x_{1_+}x_{2_+}x_{3_-}x_{4_-}}{x_{12_\perp}^2 x_{34_\perp}^2}$$

$$r \;\to\; \frac{x_{12_+}^2 (x_{3_-}x_{14_\perp}^2 - x_{4_-}x_{13_\perp}^2)^2}{x_{1_+}x_{2_+}x_{3_-}x_{4_-}x_{12_\perp}^2 x_{34_\perp}^2}$$

$$\Omega(r,\nu) \;\to\; \frac{\nu}{2\pi^2 i}\left(r^{-\frac{1}{2}+i\nu} - r^{-\frac{1}{2}-i\nu}\right)$$

Light-cone OPE in Cornalba's formula

Cornalba's formula as $x^2_{12\perp} \to 0$

$$x^4_{12}x^4_{34}\int dx_{2_+}dx_{3_-}\ \langle \mathcal{O}(L_1+x_{2_+},x_{1\perp})\mathcal{O}^\dagger(x_{2_+},x_{2\perp})\mathcal{O}(L_2+x_{3_-},x_{2\perp})\mathcal{O}^\dagger(x_{3_-},x_{3\perp})\rangle$$

$$= \frac{i\alpha_s^2}{8}\pi^2 L_1L_2\int_0^1 dudv\int d\nu\,\frac{\tanh\pi\nu}{\nu\cosh^2\pi\nu}\Big(\frac{x^2_{12\perp}x^2_{34\perp}\bar{u}u\bar{v}v}{[x^2_{13\perp}v+x^2_{14}\bar{v}]^2}\Big)^{\frac12+i\nu}\Big(\frac{L_1^2L_2^2\bar{u}u\bar{v}v}{x^2_{12\perp}x^2_{34\perp}}\Big)^{\aleph(\nu)/2}f_+$$

$$= \frac{i\alpha_s^2}{8}\pi^2\int_0^1 dv\int_{\frac12-i\infty}^{\frac12+i\infty}d\xi\, f\big(\aleph(\xi)\big), \qquad \xi\equiv\frac12+i\nu$$

$$\times\ \frac{(\bar{v}v)^{1-\xi+\frac{\aleph(\xi)}{2}}\cos\pi\xi}{(\xi-\frac12)\sin^3\pi\xi}\,\frac{B\big(2-\xi+\frac{\aleph(\xi)}{2}\big)}{[x^2_{13\perp}v+x^2_{14}\bar{v}]^{2+\aleph(\xi)}}\Big(\frac{x^2_{12\perp}x^2_{34\perp}}{[x^2_{13\perp}v+x^2_{14}\bar{v}]^2}\Big)^{-\xi-\frac{\aleph(\xi)}{2}}(L_1L_2)^{1+\aleph(\xi)}$$

Gluon light-ray (LR) operator of twist 2

$$F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$

Evolution equation (in gluodynamics)

$$\mu^2 \frac{d}{d\mu^2} F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$
$$= \int_{x_+}^{x'_+} dz'_+ \int_{x_+}^{z'_+} dz_+ \ K(x'_+, x_+; z'_+, z_+; \alpha_s) F^a_{-i}(z'_+ + x_\perp)[z'_+, z_+]^{ab} F^{b\ i}_-(z_+ + x_\perp)$$

“Forward” LR operator

$$F^\mu(L_+, x_\perp) \ = \int dx'_+ \ \left(F^a_{-i}(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab} F^{bi}_-(x_+ + x_\perp)\right)^\mu$$

Expansion in local operators

$$F^\mu(L_+, x_\perp) \ = \int_0^\infty dx_+ \sum_{n=2}^\infty \ L_+^{n-2} \mathcal{O}^g_n(x_+, x_\perp), \quad \mathcal{O}^g_n \ \equiv \ F^a_{-i} \nabla^{n-2}_- F^{ai}_-$$

Light-ray operators

Evolution equation for $F^\mu(L_+, x_\perp)$

$$\mu^2 \frac{d}{d\mu^2} F^\mu(L_+, x_\perp) = \int_0^1 du\ K_{gg}(u, \alpha_s) F^\mu(uL_+, x_\perp)$$

$$\Rightarrow \quad \gamma_n(\alpha_s) = -\frac{1}{2}\int_0^1 du\ u^{n-2} K_{gg}(u, \alpha_s) \qquad \mu\frac{d}{d\mu}\mathcal{O}^g_n = -\gamma_j(\alpha_s)\mathcal{O}^g_n$$

Conformal LR operator ($j = \frac{1}{2} + i\nu$)

$$F^\mu(L_+, x_\perp) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} (L_+)^{-\frac{3}{2}+i\nu} \mathcal{F}^\mu_{\frac{1}{2}+i\nu}(x_\perp)$$

$$\mathcal{F}^\mu_j(x_\perp) = \int_0^\infty dL_+\ L_+^{1-j} F^\mu(L_+, x_\perp)$$

Evolution equation for “forward” conformal light-ray operators

$$\Rightarrow \ \mu^2 \frac{d}{d\mu^2} \mathcal{F}^\mu_j(z_\perp) = \int_0^1 du\ K_{gg}(u, \alpha_s) u^{j-2} \mathcal{F}^\mu_j(z_\perp)$$

$\Rightarrow \gamma_j(\alpha_s)$ is an analytical continuation of γ_n

Supermultiplet of LR operators

Gluino and scalar LR operators

$$\Lambda^\mu(L_+,x_\perp) = \int dx'_+ \,\big(\bar\lambda^{a,A}(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\lambda^{b,A}(x_+ + x_\perp)\big)^\mu$$
$$\Lambda^\mu_j(x_\perp) = \int_0^\infty dL_+\, L_+^{-j}\Lambda^\mu(L_+,x_\perp)$$
$$\Phi^\mu(L_+,x_\perp) = \int dx'_+ \,\big(\bar\phi^{a,I}(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\phi^{a,I}(x_+ + x_\perp)\big)^\mu$$
$$\Phi^\mu_j(x_\perp) = \int_0^\infty dL_+\, L_+^{-j-1}\Phi^\mu(L_+,x_\perp)$$

SU_4 singlet LR operators. (non-singlet - spisat' s Grishi)

$$S^\mu_{1j}(z_\perp) = F^\mu_j(z_\perp) + \frac{j-1}{24}\Lambda^\mu_j(z_\perp) + \frac{j(j-1)}{24}\Phi^\mu(z_\perp)$$
$$S^\mu_{2j}(z_\perp) = F^\mu_j(z_\perp) - \frac{1}{24}\Lambda^\mu_j(z_\perp) - \frac{j(j+1)}{72}\Phi^\mu(z_\perp)$$
$$S^\mu_{3j}(z_\perp) = F^\mu_j(z_\perp) - \frac{j+2}{12}\Lambda^\mu_j(z_\perp) + \frac{(j+1)(j+2)}{24}\Phi^\mu(z_\perp)$$

All operators have the same anomalous dimension

$\gamma_j(\alpha_s) = \frac{2\alpha_s}{\pi}N_c[\psi(j-1)+C] + ...$

Light-cone OPE

Three-point correlator of local operators ($\mathcal{O} = \mathrm{Tr}Z^2$)

$$\langle S^\mu_{1n}(z_1)\mathcal{O}(z_2)\mathcal{O}^\dagger(z_3)\rangle = \sum \frac{c_n}{z_{12}^2 z_{13}^2 z_{23}^2}\Big(\frac{x_{12-}}{x_{12}^2} - \frac{x_{13-}}{x_{13}^2}\Big)^n \Big(\frac{\mu^{-2}x_{23}^2}{x_{12}^2 x_{13}^2}\Big)^{\frac{1}{2}\gamma(n,\alpha_s)}$$

CF of LR operator and two locals

$$x_{23}^2\int dx_{3-}\,\langle S^\mu_j(x_{1\perp})\mathcal{O}(L+x_{3-},x_{2\perp})\mathcal{O}(x_{3-},x_{3\perp})\rangle$$
$$= \int_0^1 du\,\frac{c(j,\alpha_s)[1+e^{i\pi j}]L^j(\bar u u)^j}{[x_{12\perp}^2 u + x_{13\perp}^2\bar u]^{1+j}}\Big(\frac{\bar u u \mu^{-2}x_{23\perp}^2}{[x_{12\perp}^2 u + x_{13\perp}^2\bar u]^2}\Big)^{\frac{1}{2}\gamma(j,\alpha_s)}$$

CF of two LRs : $\quad \langle S^\mu_{j=\frac{1}{2}+i\nu}(x_{1\perp})S^\mu_{j'=\frac{1}{2}+i\nu'}(x_{3\perp})\rangle = \dfrac{\delta(\nu-\nu')c(\nu,\alpha_s)}{(x_{13\perp}^2)^{j+1}(x_{13\perp}^2\mu^2)^{\gamma(j,\alpha_s)}}$

$\Rightarrow$ Light-cone OPE

$$x_{23}^2\int dx_{3-}\ \mathcal{O}(L+x_{3-},x_{2\perp})\mathcal{O}^\dagger(x_{3-},x_{3\perp})$$
$$= \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ c(j,\alpha_s)[1+e^{i\pi j}]L^j(\mu^2 x_{23\perp}^2)^{\frac{1}{2}\gamma(j,\alpha_s)}S^\mu_j(x_{3\perp})$$

DGLAP *vs* BFKL for 4-point CFs

"DGLAP" result (leading twist)

$$x_{12}^2 x_{34}^2 \int dx_{2_+} dx_{3_-}\ \langle \mathcal{O}(L_1 + x_{2_+}, x_{1\perp})\mathcal{O}^\dagger(x_{2_+}, x_{2\perp})\mathcal{O}(L_2 + x_{4_-}, x_{3\perp})\mathcal{O}^\dagger(x_{4_-}, x_{4\perp})\rangle$$

$$= \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ c(j,\alpha_s) L_1^j (\mu^2 x_{12\perp}^2)^{\frac{1}{2}\gamma(j,\alpha_s)} \langle S_j^\mu(x_{2\perp})\mathcal{O}(L_2 + x_{4_-}, x_{3\perp})\mathcal{O}^\dagger(x_{4_-}, x_{4\perp})\rangle$$

$$= \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ c(j,\alpha_s) \int_0^1 dv\ \frac{(-L_1L_2)^j}{[x_{12\perp}^2 v + x_{13\perp}^2 \bar{v}]^{1+j}} \Big(\frac{x_{12}^2 x_{34\perp}^2}{[x_{13\perp}^2 v + x_{14\perp}^2 \bar{v}]^2}\Big)^{\frac{1}{2}\gamma(j,\alpha_s)} (\bar{v}v)^{j+\frac{1}{2}\gamma(j,\alpha_s)}$$

BFKL result ($\aleph(\xi,\alpha_s) = \frac{\alpha_s N_c}{\pi}\chi(\xi) + ...$ - pomeron intercept)

$$x_{12}^2 x_{34}^2 \int dx_{2_+} dx_{3_-}\ \langle \mathcal{O}(L_1 + x_{2_+}, x_{1\perp})\mathcal{O}^\dagger(x_{2_+}, x_{2\perp})\mathcal{O}(L_2 + x_{4_-}, x_{3\perp})\mathcal{O}^\dagger(x_{4_-}, x_{4\perp})\rangle$$

$$= \int_0^1 dv \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\xi\ \frac{(\bar{v}v)^{\bar{\xi}+\frac{\aleph(\xi)}{2}} \cos\pi\xi}{(\xi - \frac{1}{2})\sin^3\pi\xi} \frac{B\big(2-\xi+\frac{\aleph(\xi)}{2}\big)}{[x_{13\perp}^2 v + x_{14}^2\bar{v}]^{2+\aleph(\xi)}} \Big[\frac{x_{12\perp}^2 x_{34\perp}^2}{[x_{13\perp}^2 v + x_{14}^2 \bar{v}]^2}\Big]^{-\xi-\frac{\aleph(\xi)}{2}} (L_1L_2)^{1+\aleph(\xi)}$$

We compare these results around $\xi \sim \omega \sim 0$

$$\Rightarrow\ 1 + \aleph(\xi,\alpha_s) = j \ \text{ and } \ \gamma(j,\alpha_s) = -\xi - \frac{\aleph(\xi)}{2}$$

$$\aleph(\xi) - 2\aleph(\xi)\aleph'(\xi) \simeq \frac{\alpha_s N_c}{\pi\xi} + \frac{\zeta(3)\alpha_s^2}{\xi} + ... \quad\rightarrow\quad \gamma_j = -2\frac{\alpha_s N_c}{\pi(j-1)} + [0 + \zeta(3)(j-1)]\Big(\frac{\alpha_s N_c}{\pi(j-1)}\Big)^3 + ...$$