HW 1. Due Tue Sept 11 at the lecture

Problem 1.

1a. A 50 ft sailboat is passing you (standing on shore) at a leisurely pace of 6 kn (3 m/s). Which of the following statements is *not* true?

1. No matter how precisely you measure, you could not find any relativistic effects. **- F**
2. The boat's clock appears to go slightly slower than your own (by about 4.5 pico-seconds / day), as measured by you **- T**
3. The boat is measured by you to be slightly shorter than 50 ft (by about 0.8 femto-meter) **- T**
4. If the crew sets off two flares, one at the bow and one at the stern, at exactly the same time (according to their reckoning), the bow one appears to be a tad later than the stern one to you (by about 0.5 femto-seconds). **- T**

1b. Which of the following statements is true?

1. A fast-moving object is "squeezed" to a shorter length by the electromagnetic interaction. **- F**
2. Fast-moving clocks go slow because the high speed messes with their internal workings. **- F**
3. Your (fast-moving) friend says you are aging more slowly than him. You claim the opposite - he is aging more slowly than you. Both he and you may be correct! **- T**
4. If the clocks at the opposite ends of a spaceship (moving away from Earth at high speed) appear slightly unsynchronized to an observer on Earth, this is because the captain did not take the finite speed of light into account when he set them. **- F**

Problem 2.

A 500 m long spaceship is moving with 90% of the speed of light relative to Earth. Every second (according to the ship's clock) two lasers send simultaneous flashes of light back to Earth - one from the front tip of the spaceship and one from the rear end. Calculate the following:

2a. According to Earth's measurement (correcting for the motion and length of the spaceship), what is the apparent time interval between the "simultaneous" (according to the space ship) emission from the tip and the rear end?

$$x_1 \;=\; \gamma(x_1' - ct'), \quad x_2 \;=\; \gamma(x_2' - ct') \quad \Rightarrow \quad x_2 - x_1 \;=\; \gamma(x_2' - x_1')$$
$$\Rightarrow \quad \Delta t \;=\; \frac{x_2 - x_1}{c} \;=\; \gamma\frac{x_2' - x_1'}{c} \;=\; \frac{10}{\sqrt{19}}\frac{500}{3\times 10^8}\mathrm{s} \;\simeq\; 3.44\times 10^{-6}s$$

2b. Using best measurement practices, what appears to be the length of the spaceship as measured from Earth?

$$l \;=\; \frac{l'}{\gamma} \;=\; 500\frac{\sqrt{19}}{10} \;\simeq\; 218\mathrm{m}$$

Problem 3.

In 2047, NASA sends the first spaceship to Alpha Centauri, the closest star outside our solar system (at 4 light-years distance). Assume the spacecraft can go at 80% of the speed of light (relative to Earth). Ignoring the (short) periods of acceleration and deceleration at each end of the trip, explain why the astronauts age by less than what an Earth-based observer would expect for the trip. (How many years would the trip take, according to Earth? How many years according to the astronauts?) How can the astronauts explain the fact that they covered a distance of 4 light- years in less than 4 years? How would THEY describe the trip, from their own reference frame?

Solution

From the numbers given, v/c = 0.8 and $\gamma = \frac{5}{3} = 1.667$. As seen from Earth, the travel time is 5 years (4 light-years divided by v = 0.8c). As we saw in lecture, “moving clocks run slow”which means that during that same 5 years, the “internal clocks” of the spacecraft and the astronauts themselves tick off only $\frac{5\text{ years}}{\gamma} \;=\; 3$ years. So they age considerably less (by only 3 years) than expected from the trip duration (5 years). But how can they explain to themselves that it took them only 3 years to cover a distance of 4 light-years? Of course, their own reference frame (that of the moving spaceship) is perfectly equivalent to Earth, so there must be a logical and consistent explanation. It comes from the fact that in their own reference frame, the whole (local) universe, including Sun, Earth and Alpha Centauri, are moving with velocity -0.8c. But moving lengths contract by a factor $\frac{1}{\gamma}$ as well! That means that the distance they have to travel *looks* to be only $\frac{4\text{ light years}}{1.667} \;=\; 2.4$ light years. This distance can indeed be traveled in only 3 years when going with velocity 0.8 c, so there is no contradiction.