HW 2. Due Tue Sept 25 at the lecture

Problem 1.

Please answer the following questions with "Y" or "N":

1a) Can a physical object have a momentum of 100 kgm/s while having a total relativistic energy of 5.109 J? **N** (since $E^2 = m^2c^4 + p^2c^2$)

1b) If two fast cars collide and form a single object, does this object have more mass (at least momentarily) than the sum of the two car masses? **Y**

1c) Two oppositely charged masses are released from rest (at large distance from each other) and after a long while crash into each other due to their electric attraction. Is the mass of the final-state object (the result of the crash) larger than the sum of the two initial masses? **N** due to conservation of energy

Problem 2.

The following is a set of multiple choice questions. Answer each with a single digit number.

2a) At the Jefferson Lab Continuous Electron Beam Accelerator, electrons are subjected to a (roughly constant) electrostatic force that increases their kinetic energy by 12×10^9 eV for each electron. (Electrons have a rest energy of 0.511×10^6 eV.) Which of the following statements is true? **2**

1 - Each electron increases its speed continuously at a constant rate $\frac{du}{dt}$. *No*

2 - At the end, each electrons has a momentum of 12.109 eV/c (within rounding errors). *Yes* (since $E_{\text{kin}} \gg mc^2$).

3 - The rest mass of each electron increases enormously during the acceleration. *No*

4 - The total energy of each electron is much larger than its kinetic energy after the acceleration. *No*, they are approximately equal

2b) What does Einstein's famous equation $E = mc^2$ imply? Pick the best answer **4**

1 - That atom bombs create a lot of mass.

2 - That it is possible to convert the mass of any object into pure energy at will. *Not at will!*

3 - That the rest mass is a conserved quantity.

4 - That a change in internal energy will also change the (apparent) mass of an object.

2c) Under what circumstances does the relativistic equation for the energy of a moving mass m agree with the non-relativistic result, $T_{\text{kin}} = \frac{1}{2}mu^2$? Pick the best answer! **3**

1 - In a coordinate system where the mass is moving with a speed much smaller than that of light.

2 - Only if you subtract or disregard the rest mass energy of the object.

3 - Both conditions above must be fulfilled.

4 - Under no circumstances.

Problem 3.

Answer the following questions with simple numbers.

3a) How fast does an object have to go to double its energy (i.e., make it twice its rest energy m_0c^2)? **A**: $\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2} \;\Rightarrow\; v = \frac{\sqrt{3}}{2}c \;\simeq\; 0.87\text{c}$

3b) A modern Li-ionbattery can run a laptop(that requires 1W average power) for 8 hours until it is completely discharged. How much less mass will it have after those 8 hours compared to its initial mass? (express your answer in kg) **A**: $3.2 \times 10^{-13}\text{kg} = \frac{28800J}{c^2}$

3c) Assume you could somehow convert 10 kg of your body mass into pure energy (wouldn?t that be great?). How long could you power your home (which requires 5kW average power) with this energy? **A**: $\frac{10\text{kg}\times c^2}{5000\text{W}} \;\simeq\; 1.8 \times 10^{14}$ sec $\simeq 5.7 \times 10^6$ years

3d) What is the rest mass of an object that has momentum 100 kgm/s and total relativitistic energy $3.16{\times}10^{10}$ J? [in kg] **A**: $m = \frac{1}{c^2}\sqrt{E^2 - p^2c^2} \;\simeq\; 10^{-7}\text{kg}$

3e) For the same object, what is its velocity? **A**: $v = \frac{pc^2}{E} \;=\; 0.95\text{c}$

3f) A subatomic particle with a rest mast of 140.106 $\frac{eV}{c^2}$ (a π^0) can decay into two equal particles, e.g. two photons or an electron plus an anti-electron (which has the same mass). How much total energy has each of the two decay particles? **A**: $|\vec{p}|$ should be the same $\Rightarrow$ $E_1 = E_2 = 70.53\frac{\text{eV}}{c^2}$

3f (*corrected*) rest mass $= 140 \times 10^6\frac{\text{eV}}{c^2} \Rightarrow$ **A**: $E_1 = E_2 = 70.53 \times 10^6\frac{\text{eV}}{c^2} \;\simeq\; 1.25 \times 10^{-18}\text{J}$

3g) How much 3-momentum does each of the two decay particles have if they are photons? How much would it be if each of them are (anti-)electrons, at a mass of 0.511.106 $\frac{eV}{c^2}$ each? **A**: For photons: $p = \frac{E}{c} = 70\frac{\text{eV}}{c} \;\simeq\; 3.73 \times 10^{-26}\frac{\text{kg}\cdot\text{m}}{\text{s}}$, for electron-positron pair $p = \sqrt{\frac{E^2}{c^2} - m^2c^2} = \sqrt{70^2 - 0.5^2}\;\frac{\text{eV}}{c} \;\simeq$ same as for photons

3g (*corrected*) For photons $p = \frac{E}{c} = 70 \times 10^6\frac{\text{eV}}{c} \simeq\; 3.73 \times 10^{-18}\frac{\text{kg}\cdot\text{m}}{\text{s}}$ and same for electrons with mass $m_e \simeq 0.5 \times 10^6\text{eV}$