HW 5 solution

Problem 1.

First thing which comes to mind is

$$(-1)^i \;=\; (e^{i\pi})^i \;=\; e^{-\pi}$$

which is real and $\Im(-1)^i = 0$.

However, this problem is an illustration of existence of *cuts* in complex functions of complex variable z. Since $a^i \equiv e^{i\ln a}$ one gets

$$(-1)^i \;=\; e^{i\ln(-1)}$$

Logarithm of (-1) is not well defined since $e^{i\pi} = e^{-i\pi} = -1$. I is said that the function $\ln z$ has a cut in the complex plane of z at real negative z and $\ln(-1) = \pm i\pi$ depending on how you approach (-1): as (-1+i0) or (-1-i0). Thus,

$$(-1)^i \;=\; e^{\mp\pi}$$

depending on how you approach (-1).

Problem 2. At time $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) \;=\; \begin{cases} A\frac{x}{a} & a \geq x \geq 0 \\ A\frac{b-x}{b-a} & b \geq x \geq a \\ 0 & \text{otherwise} \end{cases}$$

where A, a, and b are constants.

(a) Normalize Ψ (i.e., find A in terms of a and b)

(b) Where is the particle most likely to be found, at $t = 0$

(c) What is the probability of finding the particle to the left of a?

(d) What is the expectation value of x?

Solution

(a)

$$\int_0^b dx\ \Psi(x,0)|^2 \;=\; A^2\frac{a}{3} + A^2\frac{b-a}{3} \;=\; A^2\frac{b}{3} \quad\Rightarrow\quad A \;=\; \sqrt{\frac{3}{b}}$$

(b)

The function $|\Psi(x,0)|^2$ is maximal at $x = a$.

$$(\mathbf{c}) \qquad P(x \leq a) \;=\; \int_0^a dx\; |\Psi(x,0)|^2 \;=\; A^2\frac{a}{3} \;=\; \frac{a}{b}$$

(d)

$$\langle x\rangle \;=\; \int_0^b dx\; x|\Psi(x,0)|^2 \;=\; \frac{3}{b}\Big[\int_0^a dx\; x\frac{x^2}{a^2} \;+\int_a^b dx\; x\frac{(b-x)^2}{(b-a)^2}\Big] \;=\; \frac{b+2a}{4}$$

Problem 3 = Probleem 6.51.

From Eq. (6.32) we get

$$(\mathbf{a}) \qquad P(0 \leq x \leq \frac{L}{2}) \;=\; \int_0^{\frac{L}{2}} dx\; |\Psi(x,0)|^2 \;=\; \frac{2}{L}\int_0^{\frac{L}{2}} \sin^2\frac{\pi x}{L} \;=\; \frac{1}{L}\int_0^{\frac{L}{2}} dx\; [1-\cos\frac{2\pi x}{L}] \;=\; \frac{1}{2}$$

$$(\mathbf{b}) \qquad P(0 \leq x \leq \frac{L}{3}) \;=\; \int_0^{\frac{L}{3}} dx\; |\Psi(x,0)|^2 \;=\; \frac{1}{L}\int_0^{\frac{L}{3}} dx\; [1-\cos\frac{2\pi x}{L}] \;=\; \frac{1}{3}-\frac{\sin\frac{2\pi}{3}}{2\pi} \;=\; \frac{1}{3}-\frac{\sqrt{3}}{4\pi} \;\simeq\; 0.195$$

$$(\mathbf{c}) \qquad P(0 \leq x \leq \frac{3L}{4}) \;=\; \int_0^{\frac{3L}{4}} dx\; |\Psi(x,0)|^2 \;=\; \frac{1}{L}\int_0^{\frac{3L}{4}} dx\; [1-\cos\frac{2\pi x}{L}] \;=\; \frac{3}{4}-\frac{\sin\frac{3\pi}{2}}{2\pi} \;=\; \frac{3}{4}+\frac{1}{2\pi} \;\simeq\; 0.909$$