HW 6. Due Thu Nov 15 at the lecture

Problem 1.

Please answer the following questions with "Y" or "N":

Compare the ground state of the harmonic oscillator and of the square well of width L with infinitely high walls. Which properties do they have in common? Please answer the following questions with "Y" or "T" only if the statement is true for BOTH the the harmonic oscillator and the square well ground state, otherwise with "N"

1a) Both are eigenfunctions to their respective Hamilton operators.True?

1b) The full wave functions describing these states depend on time as well as on x. True?

1c) Both wave functions are identical zero outside a finite interval in x. True?

1d) The dependence of the probability density on x (i.e., the probability to find the particle in some interval in x) is unchanged over time for both. True?

Problem 2.

Let $\psi_0(x) = \psi(x, t = 0)$ be a wavefunction describing the position of a single particle along the x-axis, at time t=0. Which of the following statements is false? Answer the question with a single digit number.

1: In all cases, any solution to the Schrödinger equation can always be written as a product $\psi(x,t) = \psi_0 e^{-i\frac{E}{\hbar}t}$.

2: In general, at some arbitrary time t, the state vector must be given by $\psi(x,t)$ with $i\hbar\frac{\partial\psi(x,t)}{\partial t} = \hat{H}\psi(x,t)$ ($\hat{H}$ is the Hamiltonian operator acting on the function $\psi(x,t)$)

3: If $\psi(x,t)$ is properly normalized to 1, the probability to find the particle in the (small) interval $[x, .x + \Delta x]$ at time t is given by $|\psi(x,t)|^2\Delta x$

Problem 3.

Consider the ground state of the 1-dimensional Harmonic oscillator with mass m and frequency ω and (therefore) energy $E = \frac{\hbar\omega}{2}$

3a) Calculate the maximum amplitude A for a classical harmonic oscillator with the same energy. Express your result in terms of the constants given (ω, m)

3b) For the quantum-mechanical case, calculate the probability to find the oscillating particle within a small interval Δx around a position $x = 2A$ that is twice as far away from the origin than allowed by classical energy conservation.

Problem 4. Problem 6.51 from *Tipler & Llewellyn*, 5th ed.