# 323 final exam (16 points). 12/13/18, 12:30 p.m. -3:30 p.m.

**Problem 1**. True (**T**) or false (**F**)?

1. If two events are separated by space-like interval, there is a frame where they occur simultaneously.
2. The energy of even a massless particle like photon can be arbitrary large.
3. If I know the wave function of some quantum-mechanical system, I can predict the outcome of any future observation of that system (e.g. position) with certainty.
4. If I measure the energy of a quantum-mechanical system, I will get an answer that is an eigenvalue of the Hamiltonian of that system.
5. An atom with 12 electrons must have some electrons in $n = 3$ state.

**Problem 2**.

Two spaceships approach Earth with speeds $0.8c$. One of them goes along $x$ axis, another along $y$ axis. What is the magnitude and direction of the velocity of one of the ships in another ship's frame.

**Problem 3**.

Photons from a helium-neon laser $\lambda$=632.82 nm collide head on with incident electrons of energy $E_1$=100 MeV. Some of the photons are scattered back in the direction from which they came. What is the wavelength of the back-scattered light?

**Problem 4**.

The average energy of a proton in a certain nucleus is 20 MeV. Using Heisenberg uncertainty relation, estimate the size of the nucleus. (10 fm)

**Problem 5**.

At $t = 0$ the one-dimensional harmonic oscillator with Hamiltonian $\frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}$ is in the state

$$\frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$$

a) Show that a later time $t$ the oscillator is still in the $\frac{1}{\sqrt{2}}[\psi_0(x,t) + \psi_1(x,t)]$ state.

b) Find the average momentum $\langle \hat{p} \rangle$ at time $t$.

You may need Gaussian integrals

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \qquad \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

**Problem 6**.

The electron in a hydrogen atom is in the state described by wave function

$$\frac{1}{\sqrt{2}}(\psi_{100}e^{-i\frac{E_1}{\hbar}t} + \psi_{210}e^{-i\frac{E_2}{\hbar}t}), \qquad \psi_{nlm} \;=\; R_{nl}(r)Y_{lm}(\theta,\phi)$$

(here we disregard spin of the electron). What is the expectation value of $\hat{L}^2$ in this state at time $t$?

**Problem 7**.

What are possible values of total angular momentum for the system of three particles with spin $\frac{1}{2}$? (Assume there is no orbital angular momentum)

**Problem 8**.

Question #1 (p. 322) from *Tipler & Llewellyn, 5th ed.*

All problems have equal weight. *GOOD LUCK*!