Problem 2.1 *Calculate the approximate density of nuclear matter in gm/cm³. What would be the mass of a neutron star that had the diameter of an orange?*

Nuclear matter consists of tightly packed protons and neutrons. Therefore, to calculate the density of nuclear matter, it is sufficient to calculate the density of nucleons. As we know from Eq. (2.2) of the text, both the proton and the neutron have approximately the same mass (and their sizes are comparable) so that calculating the density of the proton is sufficient for our purpose. For the proton, we have

$$m_p \approx 938\,\mathrm{MeV}/c^2 \approx 1.67 \times 10^{-24}\,\mathrm{g}. \tag{2.1}$$

Parenthetically, we note that this leads to a relationship between the two units of energy:

$$\begin{aligned} 1\,\mathrm{erg} &= 1\,\mathrm{g} \times 1\,(\mathrm{cm/sec})^2 \\ &\approx \frac{938\,\mathrm{MeV}}{1.67 \times 10^{-24} c^2} \times 1\,(\mathrm{cm/sec})^2 \\ &\approx \frac{938 \times 10^{24}\,\mathrm{MeV}}{1.67 \times (3 \times 10^{10})^2\,\mathrm{cm^2/sec^2}} \times \mathrm{cm^2/sec^2} \\ &\approx 6.2 \times 10^5\,\mathrm{MeV} = 6.2 \times 10^{11}\,\mathrm{eV}. \end{aligned} \tag{2.2}$$

Returning to our problem, the rms "charge radius" of the proton is $\approx 0.9 \times 10^{-13}$ cm. This differs somewhat from the approximate formula given for nuclear size in Eq. (2.16) of the text, which would suggest that for $A = 1$ we have

$$R_p \approx 1.2 \times 10^{-13}\,\mathrm{cm}. \tag{2.3}$$

Treating the proton as a sphere with the above radius, the density of proton can be calculated as

$$\begin{aligned} \rho_p &= \frac{m_p}{V_p} = \frac{m_p}{\frac{4}{3}\pi R_p^3} \\ &\approx \frac{1.67 \times 10^{-24}\,\mathrm{g}}{4 \times (1.2 \times 10^{-39})^3\,\mathrm{cm^3}} \\ &\approx 2.4 \times 10^{14}\,\mathrm{g/cm^3}. \end{aligned} \tag{2.4}$$

This represents an approximate density of nuclear matter.

If we assume a diameter of a neutron star approximately the size of an orange:

$$d_{\mathrm{NS}} = 2R_{\mathrm{NS}} \approx 10\,\mathrm{cm}, \tag{2.5}$$

then its volume will be given by

$$V_{\mathrm{NS}} = \frac{4}{3}\pi R_{\mathrm{NS}}^3. \tag{2.6}$$

The mass of the neutron star would therefore approximately equal:

$$\begin{aligned} M_{\mathrm{NS}} = V_{\mathrm{NS}}\rho_p &= \frac{4}{3}\pi R_{\mathrm{NS}}^3 \times \frac{m_p}{\frac{4}{3}\pi R_p^3} \\ &= \left(\frac{R_{\mathrm{NS}}}{R_p}\right)^3 m_p \approx \left(\frac{5\,\mathrm{cm}}{1.2 \times 10^{-13}\,\mathrm{cm}}\right)^3 \times 1.67 \times 10^{-24}\,\mathrm{g} \\ &\approx 1.2 \times 10^{17}\,\mathrm{g}. \end{aligned} \tag{2.7}$$