Problem 2.6 Assume that the spin of a proton can be represented by a positive pion moving at a speed c in a circular orbit of radius 10^{-13} cm about a neutral center. Calculate the current and the magnetic moment associated with this motion. Compare this with the known magnetic moment of the proton. (*Hint:* recall that using cgs units you can write a magnetic moment $\vec{\mu} = \left(\frac{I}{c}\right)\vec{A}$, where I is the current flowing around the area A.)

For this simple model of proton motion, we have

$$\begin{aligned} e &\approx 4.8 \times 10^{-10}\,\text{esu}, \\ r &= \text{radius of circular orbit} = 10^{-13}\,\text{cm}, \\ c &= \text{speed of rotation} = 3 \times 10^{10}\,\text{cm/sec}. \end{aligned} \tag{2.25}$$

It follows that the period of rotation is given by

$$T = \frac{2\pi r}{c} \approx \frac{6 \times 10^{-13}\,\text{cm}}{3 \times 10^{10}\,\text{cm/sec}} = 2 \times 10^{-23}\,\text{sec}. \tag{2.26}$$

The current (rate of change of charge with time) associated with the motion of the charge is given by

$$I = \frac{e}{T} \approx \frac{4.8 \times 10^{-10}\,\text{esu}}{2 \times 10^{-23}\,\text{sec}} = 2.4 \times 10^{13}\,\text{esu/sec}. \tag{2.27}$$

As we know from classical electromagnetism, in CGS units, the magnetic moment associated with a circular loop carrying current I is given by

$$\mu = \frac{I}{c} \times A, \tag{2.28}$$

where A represents the area enclosed by the current loop. Applying this to the simple model of proton motion, we obtain

$$\begin{aligned} \mu_p &= \frac{I}{c} \times \pi r^2 \approx \frac{2.4 \times 10^{13}\,\text{esu/sec}}{3 \times 10^{10}\,\text{cm/sec}} \times 3 \times 10^{-26}\,\text{cm}^2 \\ &= 2.4 \times 10^{-23}\,\text{esu-cm} = 2.4 \times 10^{-23}\,\text{erg/G} \\ &\approx 2.4 \times 10^{-23} \times \frac{6.2 \times 10^{5}\,\text{MeV}}{10^{-4}\,\text{T}} \\ &\approx 1.489 \times 10^{-13}\,\text{MeV/T}, \end{aligned} \tag{2.29}$$

where we have used (2.2). The result for the magnetic moment can be compared with the measured value of the magnetic moment of the proton (given in Eq. (2.20) of the text)

$$\begin{aligned} \mu_p &\approx 2.79\mu_N = 2.79 \times \frac{m_e}{m_N} \times \mu_B \\ &\approx 2.79 \times \frac{0.511\,\text{MeV}/c^2}{938.27\,\text{MeV}/c^2} \times 5.79 \times 10^{-11}\,\text{MeV/T} \\ &\approx 2.79 \times 3.15 \times 10^{-14}\,\text{MeV/T} \approx 8.8 \times 10^{-14}\,\text{MeV/T}. \end{aligned} \tag{2.30}$$

Hence, this simple model leads to good agreement for the order of magnitude of the magnetic moment of the proton.