Weak Interactions

Classification of Weak Interactions

Type	**Comment**	**Examples**
Leptonic	involves only leptons	muon decay ($\mu \rightarrow e\nu\nu$) $\nu_e e^- \rightarrow \nu_e e^-$
Semileptonic	leptons and quarks	neutron decay ($\Delta s=0$) $K^+ \rightarrow \mu^+\nu_\mu$ ($\Delta s=1$) $B^- \rightarrow D^o\mu^-\bar{\nu}_\mu$ ($\Delta b=1$)
Non-Leptonic	involves only quarks	$\Lambda \rightarrow \pi^- p$ & $K^+ \rightarrow \pi^+\pi^o$

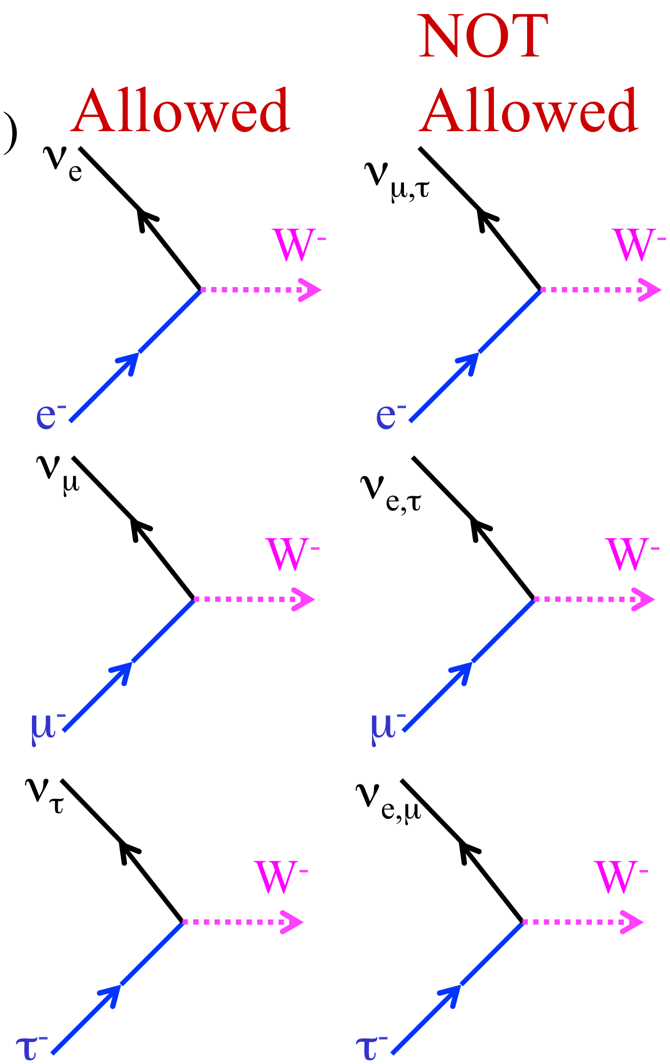
Some details of Weak Interactions

quarks and leptons are grouped into doublets (SU(2))
(sometimes called families or generations)

For every quark doublet there is a lepton doublet

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{matrix} Q = 2/3|e| \\ Q = -1/3|e| \end{matrix}$$

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \qquad \begin{matrix} Q = -|e| \\ Q = 0 \end{matrix}$$



<u>Charged Current Interactions</u> (exchange of a W boson)

W's couple to leptons in the same doublet

Cabibbo's conjecture was that the quarks that participate in the weak interaction are a <u>mixture</u> of the quarks that participate in the strong interaction.

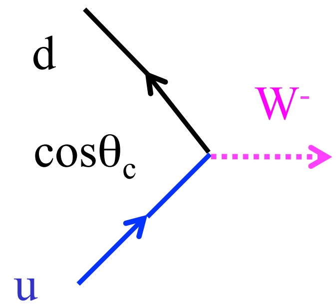
This mixing was originally postulated by Cabibbo (1963) to explain certain decay patterns in the weak interactions and originally had only to do with the *d* and *s* quarks.

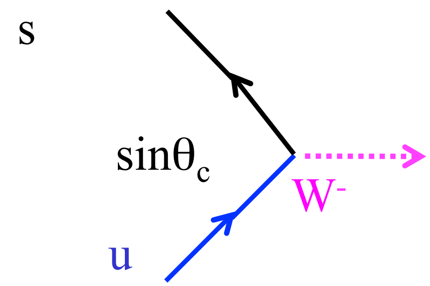
$$d' = d\cos\theta + s\sin\theta$$

Thus the form of the interaction (charged current) has an extra factor for *d* and *s* quarks

$$d \text{ quark: } J^u \;\alpha\; \gamma^u(1-\gamma^5)\cos\theta_c \qquad s \text{ quark: } J^u \;\alpha\; \gamma^u(1-\gamma^5)\sin\theta_c$$

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

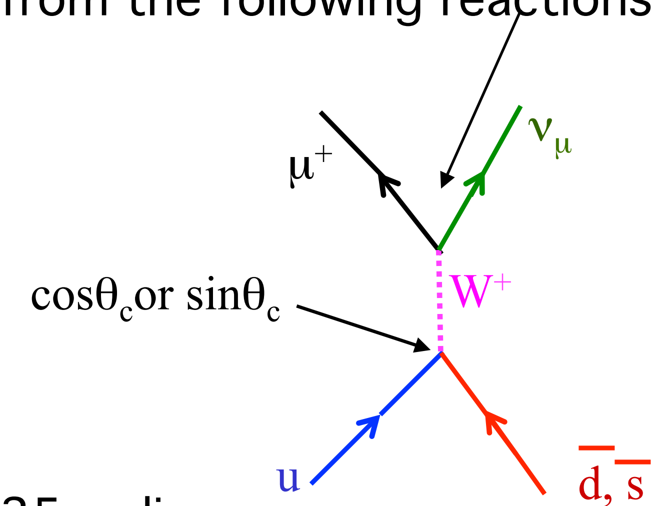




Purely leptonic decays (e.g. muon decay) do not contain the Cabibbo factor:

The Cabibbo angle is important for determining the rate of many reactions.
The Cabibbo angle can measured using data from the following reactions:

$$\frac{BR(K^+\to\mu^+\nu)}{BR(\pi^+\to\mu^+\nu)} = \frac{\sin^2\theta_c}{\cos^2\theta_c}\left[\frac{m_k}{m_\pi}\right]\left[\frac{1-(m_\mu/m_k)^2}{1-(m_\mu/m_\pi)^2}\right]^2$$



From the above branching ratio's we find:

$\theta_c = 0.27$ radians

We can check the above by measuring the rates for:

$$K^- \to \pi^o e^- \bar{\nu}_e \qquad \pi^- \to \pi^o e^- \bar{\nu}_e$$

Find: $\theta_c = 0.25$ radians

Cabibbo's Model

Extensions to the Cabibbo Model:

Cabibbo's model could easily be extended to 4 quarks:

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

$$\boxed{\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}}$$

Adding a fourth quark actually solved a long standing puzzle in weak interactions, the "absence" (i.e. very small BR) of decays involving a "flavor" (e.g. strangeness) changing neutral current:

$$\frac{BR(K^0 \to \mu^+\mu^-)}{BR(K^+ \to \mu^+\nu_\mu)} = \frac{7\times 10^{-9}}{0.64} \approx 10^{-8}$$

However, Cabibbo's model could NOT incorporate CP violation and by 1977 there was evidence for 5 quarks!

The CKM model:

In 1972 (2 years before discovery of charm!) Kobayashi and Maskawa extended Cabibbo's idea to six quarks:

- 6 quarks (3 generations or families)
- 3x3 matrix that mixes the weak quarks and the strong quarks (instead of 2x2)
- The matrix is unitary $\rightarrow$3 angles (generalized Cabibbo angles), 1 phase (instead of 1 parameter)
- **The phase allows for CP violation**

Cabibbo's name was added to make "CKM"

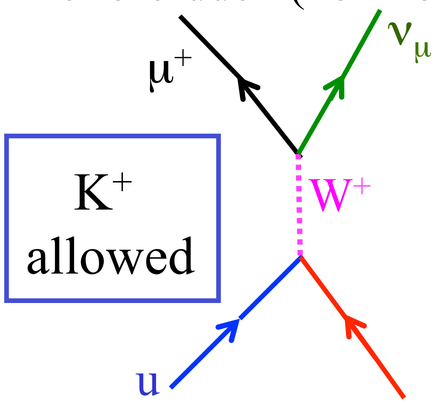
Just like θ_c had to be determined from experiment, the matrix elements of the CKM matrix must also be obtained from experiment.

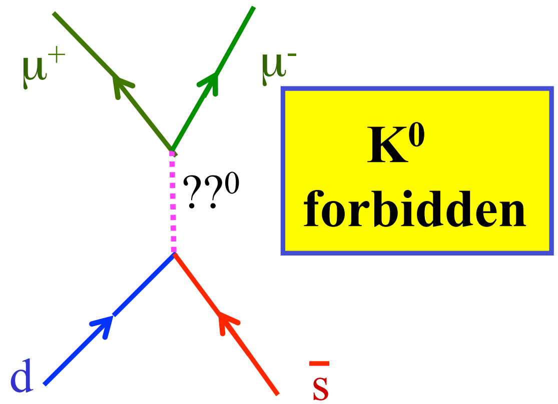
The GIM Mechanism

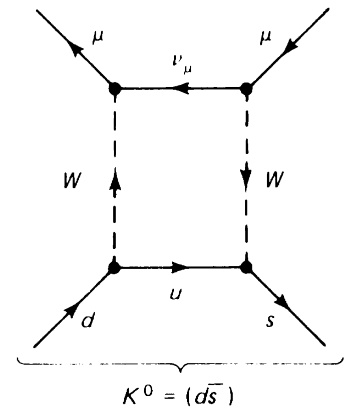
In 1969-70 Glashow, Iliopoulos, and Maiani (GIM) proposed a solution to the to the $K^0 \rightarrow \mu^+ \mu^-$ rate puzzle.

$$\frac{BR(K^0 \to \mu^+\mu^-)}{BR(K^+ \to \mu^+\nu_\mu)} = \frac{7\times10^{-9}}{0.64} \approx 10^{-8}$$

The branching fraction for $K^0 \rightarrow \mu^+ \mu^-$ was expected to be small as the first order diagram is forbidden (no allowed W coupling).







The 2nd order diagram ("box") was calculated & was found to give a rate higher than the experimental measurement!

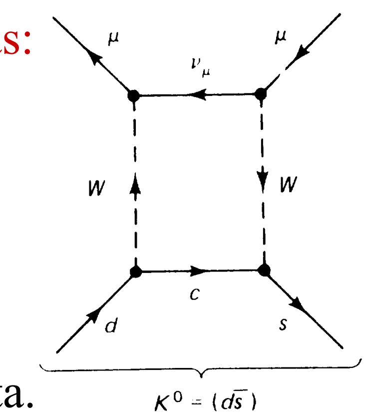
amplitude $\propto \sin\theta_c\cos\theta_c$

GIM proposed that a 4th quark existed and its coupling to the s and d quark was:

$s' = s\cos\theta - d\sin\theta$

The new quark would produce a second "box" diagram with

amplitude $\propto -\sin\theta_c\cos\theta_c$



These two diagrams almost cancel each other out.

The amount of cancellation depends on the mass of the new quark

A quark mass of ≈1.5GeV is necessary to get good agreement with the exp. data.

First "evidence" for Charm quark!

CKM Matrix

The CKM matrix can be written in many forms:

1) In terms of three angles and phase:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This matrix is not unique, many other 3X3 forms in the literature. This one is from PDG2000.

The four real parameters are δ, θ_{12}, θ_{23}, and θ_{13}.

Here s=sin, c=cos, and the numbers refer to the quark generations, e.g. $s_{12}=\sin\theta_{12}$.

2) In terms of coupling to charge 2/3 quarks (best for illustrating physics!)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

3) In terms of the sine of the Cabibbo angle (θ_{12}).

This representation uses the fact that $s_{12} >> s_{23} >> s_{13}$.

"Wolfenstein" representaton

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Here $\lambda=\sin\theta_{12}$, and A, ρ, η are all real and approximately one.

This representation is very good for relating CP violation to specific decay rates.

CKM Matrix

The magnitudes of the CKM elements, from experiment are (PDG2000):

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & (2-5)\times 10^{-3} \\ 0.219-0.225 & 0.9734-0.9749 & (3.7-4.3)\times 10^{-2} \\ (0.4-1.4)\times 10^{-2} & (3.5-4.3)\times 10^{-2} & 0.9990-0.9993 \end{pmatrix}$$

There are several interesting patterns here:

1) The CKM matrix is almost diagonal (off diagonal elements are small).
2) The further away from a family, the smaller the matrix element (e.g. $V_{ub} << V_{ud}$).
3) Using 1) and 2), we see that certain decay chains are preferred:
   $c \rightarrow s$ over $c \rightarrow d$ $\quad D^0 \rightarrow K^-\pi^+$ over $D^0 \rightarrow \pi^-\pi^+$ (exp. find 3.8% vs 0.15%)
   $b \rightarrow c$ over $b \rightarrow u$ $\quad B^0 \rightarrow D^-\pi^+$ over $B^0 \rightarrow \pi^-\pi^+$ (exp. find 3×10^{-3} vs 1×10^{-5})
4) Since the matrix is supposed to be unitary there are lots of constraints among the matrix elements:

$$V_{ud}^* V_{ud} + V_{cd}^* V_{cd} + V_{td}^* V_{td} = 1$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

So far experimental results are consistent with expectations from a Unitary matrix. But as precision of experiments increases, we *might* see deviations from Unitarity.

Measuring the CKM Matrix

No one knows how to calculate the values of the CKM matrix.

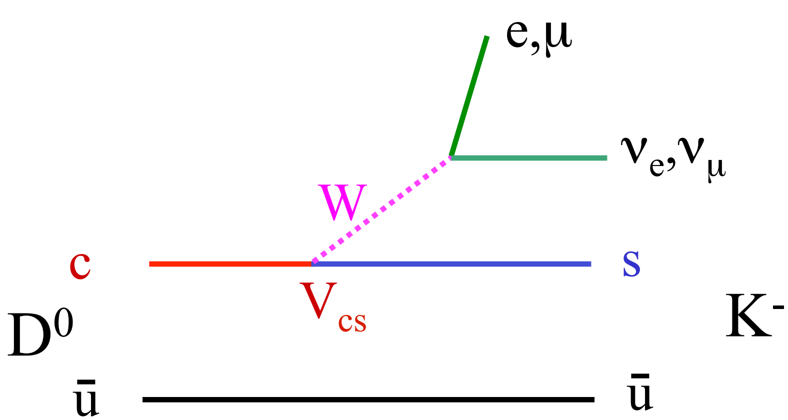
Experimentally, the cleanest way to measure the CKM elements is by using interactions or decays involving leptons.

$\Rightarrow$ CKM factors are only present at one vertex in decays with leptons.

V_{ud}: neutron decay:	$n \to pe\nu$	$d \to ue\nu$
V_{us}: kaon decay:	$K^0 \to \pi^+ \bar{e}^- \nu_e$	$s \to ue\nu$
V_{bu}: B-meson decay:	$B^- \to (\rho \text{ or } \pi^+) \bar{e}^- \nu_e$	$b \to ue\nu$
V_{bc}: B-meson decay:	$B^- \to D^0 \bar{e}^- \nu_e$	$b \to ce\nu$
V_{cs}: charm decay:	$D^0 \to K^- e^+ \nu_e$	$c \to se\nu$
V_{cd}: neutrino interactions:	$\nu_\mu d \to \mu^- c$	$d \to c$

For massless neutrinos the lepton "CKM" matrix is diagonal

"Spectator" Model decay of $D^0 \to K^- e^+ \nu_e$



Amplitude $\propto V_{cs}$

Decay rate $\propto |V_{cs}|^2$

Called a "spectator" diagram because only one quark participates in the decay, the other "stands around and watches".

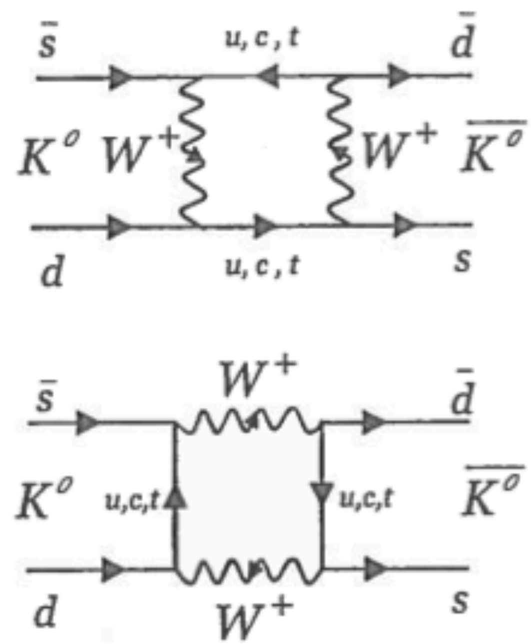


Fig. 14.6 The $\Delta S = 2$ transition "box diagrams" via two consecutive weak processes that are responsible for K^0-$\overline{K^0}$ mixing and indirect CP violation in K^0 decay.

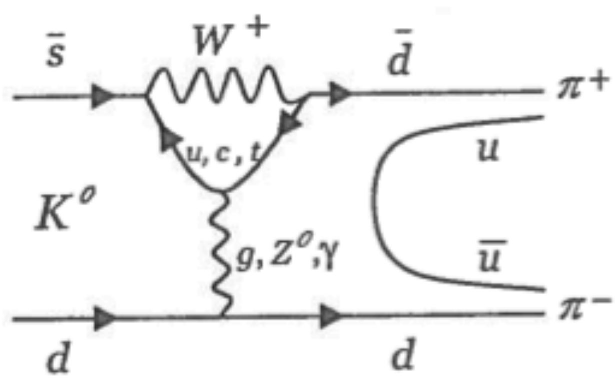


Fig. 14.7 The $\Delta S = 1$ transition, or "penguin diagram", that is responsible for direct CP violation in K^0 decay.