

preserving charge conjugation either for $C = +$ or $C = -$ states. (Again, the reverse reaction can only proceed virtually.)

Problem 11.3 If ρ^0 mesons are produced in states with spin projection $J_z = 0$ along their line of flight, what would you expect for the angular distribution of $\rho^0 \rightarrow \pi^+ + \pi^-$ decay products in the ρ^0 rest frame? (See Appendix B for the appropriate $Y_{\ell,m}(\theta, \phi)$ functions.) What would be your answer if the initial ρ^0 had spin projection $J_z = +1$?

Let us assume that the line of flight of the produced ρ^0 meson defines the z -axis. In the rest frame of the ρ^0 meson, the π^+ and π^- mesons will be produced back-to-back in order to conserve momentum.

The spin-parity quantum numbers in the strong decay

$$\rho^0 \rightarrow \pi^+ + \pi^-, \quad (11.15)$$

are

$$1^- \rightarrow 0^- + 0^-. \quad (11.16)$$

Thus, we see that conservation of angular momentum (as well as parity, since it is a strong process) requires that the relative orbital angular momentum of the final state be

$$\ell = 1. \quad (11.17)$$

Because pions are spin-zero particles, this represents the total angular momentum of the final state.

(a) If the initial ρ^0 meson has $J_z = 0$, then conservation of angular momentum (projection) requires that the final state of two pions must have

$$\ell_z = m = 0. \quad (11.18)$$

The spatial component of the wave function will have the form

$$\psi_{\pi^+\pi^-}(r, \theta, \phi) \approx f(r)Y_{10}(\theta, \phi) \approx f(r)\cos\theta, \quad (11.19)$$

where $f(r)$ is the radial component of the wave function, which is not relevant for our discussion. The form of the spherical harmonics is from Eq. (B.6) of the text. (The isospin component of this decay has

already been discussed in Problem 11.1.) The angular distribution of the decay products is therefore

$$\frac{d\sigma}{d\Omega} \sim |\psi_{\pi^+\pi^-}(r, \theta, \phi)|^2 \sim |Y_{10}(\theta, \phi)|^2 \sim \cos^2\theta. \quad (11.20)$$

(b) If the decaying ρ^0 meson is in the spin state $J_z = \pm 1$, then conservation of angular momentum implies that the final state should have

$$\ell_z = m = \pm 1. \quad (11.21)$$

The final state wave function will have the form

$$\psi_{\pi^+\pi^-}(r, \theta, \phi) = f(r)Y_{1,\pm 1}(\theta, \phi) \sim f(r)\sin\theta e^{\pm i\phi}. \quad (11.22)$$

This leads to an angular distribution of the form

$$\frac{d\sigma}{d\Omega} \sim |\psi_{\pi^+\pi^-}(r, \theta, \phi)|^2 \sim |Y_{1,\pm 1}(\theta, \phi)|^2 \sim \sin^2\theta. \quad (11.23)$$

The angular distributions in Eqs. (11.20) and (11.23) can be distinguished in data to provide the J_z of the original ρ meson, and thereby yield information on its production mechanism.

Problem 11.4 The Ξ^- has $J^P = \frac{1}{2}^+$. It decays through weak interaction into a Λ^0 and a π^- meson. If $J_\Lambda^P = \frac{1}{2}^+$ and $J_\pi^P = 0^-$, what are the allowed relative orbital angular momenta for the $\Lambda-\pi^-$ system?

The reaction

$$\Xi^- \rightarrow \Lambda^0 + \pi^-, \quad (11.24)$$

is a weak decay process. We therefore expect several quantum numbers to be violated. Looking at Table 9.4 of the text, it is clear that this process violates both isospin and strangeness quantum numbers,