The α particle has a rest mass given approximately by

$$M = 4 \times 10^3 \,\text{MeV}/c^2. \tag{1.46}$$

If the α particle has a kinetic energy

$$T = 4\,\text{MeV}, \tag{1.47}$$

and if we treat it nonrelativistically, then we have

$$\begin{aligned} T = \frac{1}{2}Mv_{\text{NR}}^2 &= \frac{1}{2}Mc^2 \times \left(\frac{v_{\text{NR}}}{c}\right)^2 = 4 \text{ MeV} \\ \text{or} \quad \frac{1}{2} \times 4 \times 10^3 \text{ MeV} &\times \left(\frac{v_{\text{NR}}}{c}\right)^2 = 4 \text{ MeV} \\ \text{or} \quad \left(\frac{v_{\text{NR}}}{c}\right)^2 &= 2 \times 10^{-3} \\ \text{or} \quad \frac{v_{\text{NR}}}{c} &= \sqrt{20} \times 10^{-2} \approx 0.045. \end{aligned} \tag{1.48}$$

Here v_{NR} represents the magnitude of the velocity of the nonrelativistic particle.

On the other hand, if we treat the α particle as relativistic, we can then use the relativistic relationships from Eqs. (A.7) and (A.10) of Appendix A of the text to write

$$\begin{aligned} E &= \gamma Mc^2, \\ T &= E - Mc^2 = (\gamma - 1)Mc^2, \\ cP &= \sqrt{T^2 + 2Mc^2T} = \sqrt{\gamma^2 - 1}Mc^2, \end{aligned} \tag{1.49}$$

where P is the magnitude of the momentum. The relativistic velocity now follows using Eq. (A.8) of the text

$$\frac{v_{\text{R}}}{c} = \frac{cP}{E} = \frac{\sqrt{(\gamma + 1)(\gamma - 1)}}{\gamma}. \tag{1.50}$$

From the fact that the α particle has kinetic energy

$$T = 4\,\text{MeV}, \tag{1.51}$$

we can determine the Lorentz factor using (1.49)

$$\begin{aligned} T &= (\gamma - 1)Mc^2 = 4\,\text{MeV} \\ \text{or} \quad (\gamma - 1) &\times 4 \times 10^3 \text{ MeV} = 4\,\text{MeV} \\ \text{or} \quad \gamma &= 1 + 10^{-3}. \end{aligned} \tag{1.52}$$

Using this in (1.50), we can determine the relativistic velocity

$$\begin{aligned} \frac{v_{\text{R}}}{c} &= \frac{\sqrt{(2 + 10^{-3})10^{-3}}}{1 + 10^{-3}} \\ &\approx \sqrt{20} \times 10^{-2}(1 + 0.5 \times 10^{-3})^{1/2}(1 - 10^{-3}) \\ &\approx \sqrt{20} \times 10^{-2}(1 + 0.025 \times 10^{-3})(1 - 10^{-3}) \\ &\approx \sqrt{20} \times 10^{-2}(1 - 0.00075). \end{aligned} \tag{1.53}$$

Using (1.48) we see that we can write the relativistic velocity in Eq. (1.53) as

$$v_{\text{R}} = v_{\text{NR}}(1 - 0.00075). \tag{1.54}$$

Consequently, we can define the relative error in neglecting relativity as

$$\frac{|\Delta v|}{v_{\text{NR}}} = \frac{|v_{\text{R}} - v_{\text{NR}}|}{v_{\text{NR}}} \approx 0.00075 = 0.07\%. \tag{1.55}$$

For the scattering of such an α particle from gold (Au), the distance of closest approach can be dertermined as follows. First we note that the distance of closest approach is attained when the impact parameter vanishes (for head on collisions). From Eq. (1.25) of the text, we see that the distance of closest in this case ($b = 0$) is given by

$$r_0 = \frac{ZZ'e^2}{E}, \tag{1.56}$$

where, for scattering of α particles from gold (Au), we have

$$Z = 2, \quad Z' = 79. \tag{1.57}$$

If we treat the α particle nonrelativistically, we have

$$E = T = 4\,\text{MeV}. \tag{1.58}$$

Using all of these, we determine

$$\begin{aligned} r_0 &= ZZ' \times \frac{\hbar c}{E} \times \frac{e^2}{\hbar c} \\ &= 2 \times 79 \times \frac{197 \text{ MeV} - \text{F}}{4 \text{ MeV}} \times \frac{1}{137} \\ &\approx 56 \text{ F} = 5.6 \times 10^{-12} \text{ cm}. \end{aligned} \tag{1.59}$$