

which does not agree with the observed value of

$$\mu_{\text{obs}} = 0.8\mu_N. \quad (3.28)$$

$^{41}\text{Ca}^{20}$  has 20 paired neutrons, 20 paired protons, and one unpaired neutron. The shell structure has the form

$$\begin{aligned} n &: (1S_{1/2})^2(1P_{3/2})^4(1P_{1/2})^2(1D_{5/2})^6(2S_{1/2})^2(1D_{3/2})^4(1F_{7/2})^1, \\ p &: (1S_{1/2})^2(1P_{3/2})^4(1P_{1/2})^2(1D_{5/2})^6(2S_{1/2})^2(1D_{3/2})^4. \end{aligned}$$

The last neutron in the state  $1F_{7/2}$  determines

$$J^P = \frac{7}{2}^-, \quad (3.29)$$

consistent with experiment. The predicted value of the magnetic moment is that of the unpaired neutron

$$\mu = -1.91\mu_N, \quad (3.30)$$

which differs somewhat from the observed value of

$$\mu_{\text{obs}} = -1.6\mu_N. \quad (3.31)$$

**Problem 3.6** Consider a somewhat more sophisticated model for the anomalous contribution to the magnetic moment of a nucleon. Assume that the proton can be regarded as a fixed neutral center with a  $\pi^+$  meson circling about in an  $\ell = 1$  orbit. Similarly, take a neutron as an effective proton center with a  $\pi^-$  meson in an  $\ell = 1$  orbit around it. Using  $m_\pi = 140 \text{ MeV}/c^2$ , calculate  $\mu = \left(\frac{e\hbar}{2m_\pi c}\right)\ell$ , and compare results with those of Problem 2.5.

If we assume such an “atomic” model for the nucleons, then the magnetic moment of the  $\pi$  meson will be given by

$$\mu_\pi = \left(\frac{e\hbar}{2m_\pi c}\right)\ell, \quad (3.32)$$

where  $e$  represents the charge of the pion, the mass of the pion is given by

$$m_{\pi^+} = m_{\pi^-} = 140 \text{ MeV}/c^2, \quad (3.33)$$

and  $\ell$  represents the orbital angular momentum of the pion. Since the  $\pi^\pm$  mesons move in orbits with  $\ell = 1$ , we obtain

$$\begin{aligned} \mu_{\pi^\pm} &= \left(\frac{e\hbar}{2m_{\pi^\pm}c}\right) \times 1 = \pm \frac{m_p}{m_{\pi^\pm}} \mu_N \\ &= \pm \frac{938.27 \text{ MeV}/c^2}{140 \text{ MeV}/c^2} \times \mu_N \approx \pm 6.7\mu_N. \end{aligned} \quad (3.34)$$

In the model for the proton, where we assume that a  $\pi^+$  is going around a neutron, we can predict

$$\begin{aligned} \mu_p &= \mu_n + \mu_{\pi^+} \approx (-1.91 + 6.7)\mu_N = 4.79\mu_N \\ &\approx 4.79 \times 3.15 \times 10^{-14} \text{ MeV/T} \approx 1.51 \times 10^{-13} \text{ MeV/T}, \end{aligned} \quad (3.35)$$

where we have used the value of  $\mu_N$  in MeV/T from Eq. (2.30). This is quite comparable to the result in Problem 2.5.

For the neutron, the model assumes that a  $\pi^-$  moves around a stationary proton so that we have

$$\begin{aligned} \mu_n &= \mu_p + \mu_{\pi^-} \approx (2.79 - 6.7)\mu_N = -3.91\mu_N \\ &\approx -3.91 \times 3.15 \times 10^{-14} \text{ MeV/T} \approx -1.23 \times 10^{-13} \text{ MeV/T}. \end{aligned} \quad (3.36)$$

**Problem 3.7** The ground state of  $^{137}\text{Ba}^{56}$  has spin-parity  $\frac{3}{2}^+$ . That is, its spin is  $\frac{3}{2}$  and parity +. The first two excited states have spin parity  $\frac{1}{2}^+$  and  $\frac{11}{2}^-$ . According to the shell model, what assignments would be expected for these excited states? (Hint: The surprise has to do with “pairing energy”.)

$^{137}\text{Ba}^{56}$  has 56 protons and 81 neutrons. The protons are all paired and therefore do not contribute to the spin parity. According to the single-particle shell model, the neutrons should fill the energy levels