4. Nuclear Radiation

Problem 4.1 *Calculate the Q values for the following α-decays between ground-state levels of the nuclei:* (a) $^{208}Po \rightarrow {}^{204}Pb + \alpha$ *and* (b) $^{230}Th \rightarrow {}^{226}Ra + \alpha$. *What are the kinetic energies of the α-particles and of the nuclei in the final state if the decays proceed from rest?*

From the *CRC Handbook* we have the atomic masses

$$\begin{aligned} M({}^{208}\mathrm{Po}^{84}) &= 207.9812\,\mathrm{amu}, \quad M({}^{204}\mathrm{Pb}^{82}) = 203.9730\,\mathrm{amu}, \\ M({}^{230}\mathrm{Th}^{90}) &= 230.0331\,\mathrm{amu}, \quad M({}^{226}\mathrm{Ra}^{88}) = 226.0254\,\mathrm{amu}, \\ M({}^{4}\mathrm{He}^{2}) &= 4.0026\,\mathrm{amu}. \end{aligned} \tag{4.1}$$

From Eq. (4.4) of the text, the Q value in a reaction involving α decay is given by

$$Q = T_D + T_\alpha = (M(A,Z) - M(A-4, Z-2) - M(4,2))c^2, \tag{4.2}$$

where we assume that $M(A,Z)$ and $M(A-4,Z-2)$ represent the masses of the parent and the daughter nuclei (atomic masses can be used because the masses of the electrons cancel out). Furthermore, the kinetic energies of the α particle and the daughter nuclei are

$$T_\alpha = \frac{M_D}{M_D + M_\alpha} = \frac{M(A-4, Z-2)}{M(A-4,Z-2) + M(4,2)}, \quad T_D = Q - T_\alpha. \tag{4.3}$$

With all this information, we can look at the reaction

$$^{208}\mathrm{Po}^{84} \rightarrow {}^{204}\mathrm{Pb}^{82} + \alpha, \tag{4.4}$$

and we have

$$\begin{aligned} Q &= \left(M({}^{208}\mathrm{Po}^{84}) - M({}^{204}\mathrm{Pb}^{82}) - M({}^{4}\mathrm{He}^{2})\right)c^2 \\ &\approx (207.9812 - 203.9730 - 4.0026)\,\mathrm{amu} \times c^2 \\ &\approx 0.0056 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 = 5.2164\,\mathrm{MeV}, \\ T_\alpha &= \frac{M({}^{204}\mathrm{Pb}^{82})}{M({}^{204}\mathrm{Pb}^{82}) + M({}^{4}\mathrm{He}^{2})} \\ &\approx \frac{203.9730\,\mathrm{amu}}{(203.9730 + 4.0026)\,\mathrm{amu}} \times 5.2164\,\mathrm{MeV} \\ &\approx 0.98 \times 5.2164\,\mathrm{MeV} \approx 5.11\,\mathrm{MeV}, \\ T_D &= Q - T_\alpha \approx (5.2164 - 5.11)\,\mathrm{MeV} \approx 0.11\,\mathrm{MeV}, \end{aligned} \tag{4.5}$$

where in the intermediate steps we have used Eq. (2.11), which relates the "amu" unit to the "MeV" unit.

Similarly, for the reaction

$$^{230}\mathrm{Th}^{90} \rightarrow {}^{226}\mathrm{Ra}^{88} + \alpha, \tag{4.6}$$

we have

$$\begin{aligned} Q &= \left(M({}^{230}\mathrm{Th}^{90}) - M({}^{226}\mathrm{Ra}^{88}) - M({}^{4}\mathrm{He}^{2})\right)c^2 \\ &\approx (230.0331 - 226.0254 - 4.0026)\,\mathrm{amu} \times c^2 \\ &\approx 0.0051 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 \approx 4.7506\,\mathrm{MeV}, \\ T_\alpha &= \frac{M({}^{226}\mathrm{Ra}^{88})}{M({}^{226}\mathrm{Ra}^{88}) + M({}^{4}\mathrm{He}^{2})} \\ &\approx \frac{226.0254\,\mathrm{amu}}{(226.0254 + 4.0026)\,\mathrm{amu}} \times 4.7506\,\mathrm{MeV} \\ &\approx 0.982 \times 4.7506\,\mathrm{MeV} \approx 4.66\,\mathrm{MeV}, \\ T_D &= Q - T_\alpha \approx (4.7506 - 4.66)\,\mathrm{MeV} \approx 0.09\,\mathrm{MeV}. \end{aligned} \tag{4.7}$$

Problem 4.2 *Estimate the relative contribution of the centrifugal barrier and the Coulomb barrier in the scattering of a 4 MeV α-particle from ^{236}U. In particular, consider impact parameters of $b = 1\,\mathrm{fm}$ and $b = 7\,\mathrm{fm}$. What are the orbital quantum numbers in such collisions. (Hint: $|\vec{L}| \sim |\vec{r} \times \vec{p}| \sim \hbar k b \sim \hbar \ell$.)*

The scattering of an α particle from a $^{236}\mathrm{U}^{92}$ nucleus is governed by a Schrödinger equation of the kind given in Eq. (3.28) of the text,