

## 4. Nuclear Radiation

**Problem 4.1** Calculate the  $Q$  values for the following  $\alpha$ -decays between ground-state levels of the nuclei: (a)  $^{208}\text{Po} \rightarrow ^{204}\text{Pb} + \alpha$  and (b)  $^{230}\text{Th} \rightarrow ^{226}\text{Ra} + \alpha$ . What are the kinetic energies of the  $\alpha$ -particles and of the nuclei in the final state if the decays proceed from rest?

From the *CRC Handbook* we have the atomic masses

$$\begin{aligned} M(^{208}\text{Po}^{84}) &= 207.9812 \text{ amu}, & M(^{204}\text{Pb}^{82}) &= 203.9730 \text{ amu}, \\ M(^{230}\text{Th}^{90}) &= 230.0331 \text{ amu}, & M(^{226}\text{Ra}^{88}) &= 226.0254 \text{ amu}, \\ M(^4\text{He}^2) &= 4.0026 \text{ amu}. \end{aligned} \quad (4.1)$$

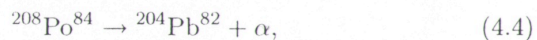
From Eq. (4.4) of the text, the  $Q$  value in a reaction involving  $\alpha$  decay is given by

$$Q = T_D + T_\alpha = (M(A, Z) - M(A - 4, Z - 2) - M(4, 2))c^2, \quad (4.2)$$

where we assume that  $M(A, Z)$  and  $M(A - 4, Z - 2)$  represent the masses of the parent and the daughter nuclei (atomic masses can be used because the masses of the electrons cancel out). Furthermore, the kinetic energies of the  $\alpha$  particle and the daughter nuclei are

$$T_\alpha = \frac{M_D}{M_D + M_\alpha} = \frac{M(A - 4, Z - 2)}{M(A - 4, Z - 2) + M(4, 2)}, \quad T_D = Q - T_\alpha. \quad (4.3)$$

With all this information, we can look at the reaction

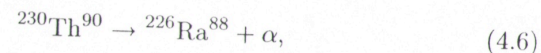


and we have

$$\begin{aligned} Q &= (M(^{208}\text{Po}^{84}) - M(^{204}\text{Pb}^{82}) - M(^4\text{He}^2))c^2 \\ &\approx (207.9812 - 203.9730 - 4.0026) \text{ amu} \times c^2 \\ &\approx 0.0056 \times 931.5 \text{ MeV}/c^2 \times c^2 = 5.2164 \text{ MeV}, \\ T_\alpha &= \frac{M(^{204}\text{Pb}^{82})}{M(^{204}\text{Pb}^{82}) + M(^4\text{He}^2)} \\ &\approx \frac{203.9730 \text{ amu}}{(203.9730 + 4.0026) \text{ amu}} \times 5.2164 \text{ MeV} \\ &\approx 0.98 \times 5.2164 \text{ MeV} \approx 5.11 \text{ MeV}, \\ T_D &= Q - T_\alpha \approx (5.2164 - 5.11) \text{ MeV} \approx 0.11 \text{ MeV}, \end{aligned} \quad (4.5)$$

where in the intermediate steps we have used Eq. (2.11), which relates the “amu” unit to the “MeV” unit.

Similarly, for the reaction



we have

$$\begin{aligned} Q &= (M(^{230}\text{Th}^{90}) - M(^{226}\text{Ra}^{88}) - M(^4\text{He}^2))c^2 \\ &\approx (230.0331 - 226.0254 - 4.0026) \text{ amu} \times c^2 \\ &\approx 0.0051 \times 931.5 \text{ MeV}/c^2 \times c^2 \approx 4.7506 \text{ MeV}, \\ T_\alpha &= \frac{M(^{226}\text{Ra}^{88})}{M(^{226}\text{Ra}^{88}) + M(^4\text{He}^2)} \\ &\approx \frac{226.0254 \text{ amu}}{(226.0254 + 4.0026) \text{ amu}} \times 4.7506 \text{ MeV} \\ &\approx 0.982 \times 4.7506 \text{ MeV} \approx 4.66 \text{ MeV}, \\ T_D &= Q - T_\alpha \approx (4.7506 - 4.66) \text{ MeV} \approx 0.09 \text{ MeV}. \end{aligned} \quad (4.7)$$

**Problem 4.2** Estimate the relative contribution of the centrifugal barrier and the Coulomb barrier in the scattering of a 4 MeV  $\alpha$ -particle from  $^{236}\text{U}$ . In particular, consider impact parameters of  $b = 1 \text{ fm}$  and  $b = 7 \text{ fm}$ . What are the orbital quantum numbers in such collisions. (Hint:  $|\vec{L}| \sim |\vec{r} \times \vec{p}| \sim \hbar kb \sim \hbar l$ .)

The scattering of an  $\alpha$  particle from a  $^{236}\text{U}^{92}$  nucleus is governed by a Schrödinger equation of the kind given in Eq. (3.28) of the text,