obtained more simply. Note that a change by a factor of 10 in $N(t)$ corresponds to a change of 1 in $\ln \Delta N(t)$. The slope in log (not ln) can therefore be read off the graph! (A least-square fit to the data is also shown in the plot.)

The slope calculated from the data gives an estimate of

$$\lambda \approx 3.1 \times 10^{-3}\,\text{min}^{-1} \tag{5.23}$$

which, in turn, leads to

$$\begin{aligned} \tau &= \text{mean life} = \frac{1}{\lambda} \approx 322\,\text{min}, \\ t_{1/2} &= \text{half life} = \tau \ln 2 \approx 224\,\text{min}. \end{aligned} \tag{5.24}$$

The statistical uncertainties shown in Fig. 5.1 correspond to square roots in the number of events (Poisson statistics). The fit is reliable since 7 of the 8 points lie within one standard deviation (error bar) of the straight line fitted to the data.

Problem 5.4 *A relic from an Egyptian tomb contains* 1 gm *of carbon with a measured activity of* 4×10^{-12} Ci. *If the ratio of* $\frac{^{14}C}{^{12}C}$ *nuclei in a live tree is* 1.3×10^{-12}, *how old is the relic? Assume the half-life of* ^{14}C *is* 5730 yr.

We know from the previous problem (as well as from Eq. (5.26) of the text) that

$$\mathcal{A}(t) = \mathcal{A}(0)e^{-\lambda t}, \quad \mathcal{A}(0) = \lambda N(0), \tag{5.25}$$

where the decay constant λ is related to the half-life as

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}. \tag{5.26}$$

For the present problem, we are given that

$$\begin{aligned} t_{1/2}^{(^{14}C)} &= 5730\,\text{yr} = 5730 \times 365 \times 24 \times 60 \times 60\,\text{sec} \\ &\approx 1.8 \times 10^{11}\,\text{sec}, \\ \lambda^{(^{14}C)} &= \frac{0.693}{t_{1/2}^{^{14}C}} \approx \frac{0.693}{1.8 \times 10^{11}\,\text{sec}} \\ &\approx 3.8 \times 10^{-12}/\text{sec}. \end{aligned} \tag{5.27}$$

Since the ratio of $\frac{^{14}C}{^{12}C}$ nuclei in a living tree is given as 1.3×10^{-12}, in 1 g of carbon, the number of ^{14}C nuclei is given by

$$N_{(^{14}C)} \approx 1.3 \times 10^{-12} \times \frac{6 \times 10^{23}}{12} = 6.5 \times 10^{10}. \tag{5.28}$$

It follows therefore that

$$\begin{aligned} \mathcal{A}(0) &= \lambda^{(^{14}C)} N_{(^{14}C)}(0) \approx 3.8 \times 10^{-12}/\text{sec} \times 6.5 \times 10^{10}\,\text{decays} \\ &\approx 0.25\,\text{decays/sec}. \end{aligned} \tag{5.29}$$

The present activity of the relic is

$$\begin{aligned} \mathcal{A}(t) &= 4 \times 10^{-12}\,\text{Ci} = 4 \times 10^{-12} \times 3.7 \times 10^{10}\,\text{decays/sec} \\ &\approx 0.15\,\text{decays/sec}, \end{aligned} \tag{5.30}$$

where we have used the definition of Curie given in Eq. (5.29) of the text.

Using these values, we determine from Eq. (5.25) that

$$\begin{aligned} -\lambda^{(^{14}C)} t &= \ln \frac{\mathcal{A}(t)}{\mathcal{A}(0)} \approx \ln \frac{0.15}{0.25} \approx -0.51 \\ \text{or} \quad t &\approx \frac{0.51}{\lambda^{(^{14}C)}} \approx \frac{0.51}{3.8 \times 10^{-12}/\text{sec}} \approx 1.3 \times 10^{11}\,\text{sec} \\ &\approx \frac{1.3 \times 10^{11}\,\text{sec}}{3.1 \times 10^{7}\,\text{sec/yr}} \approx 4193\,\text{yrs}. \end{aligned} \tag{5.31}$$

Thus, the relic is approximately 4193 yrs old.

Problem 5.5 *If the lifetime of the proton is* 10^{33} yr, *how many proton decays would you expect per year in a mass of* 10^3 metric tons *of water? What would be the approximate number expected in the year* 2050?

If the mean life of the proton is

$$\tau_p = 10^{33}\,\text{yr} \approx 10^{33} \times 3.1 \times 10^{7}\,\text{sec} = 3.1 \times 10^{40}\,\text{sec}, \tag{5.32}$$

then its decay constant is

$$\lambda_p = \frac{1}{\tau_p} = 10^{-33}/\text{yr} \approx \frac{10^{-33}}{3.1 \times 10^{7}\,\text{sec}} \approx 3.2 \times 10^{-41}/\text{sec}. \tag{5.33}$$

This is extremely small.