Considering an electron of 4 MeV (kinetic energy), the momentum is relativistic, and

$$p = \frac{1}{c}\sqrt{E^2 - m_e^2c^4} = \frac{1}{c}\sqrt{(T + m_ec^2)^2 - m_e^2c^4}$$
$$= \frac{1}{c}\sqrt{T^2 + 2m_ec^2T} = \frac{T}{c}\sqrt{1 + \frac{2m_ec^2}{T}},$$

$$p_e = (0.004\,\text{GeV}/c)\sqrt{1 + \frac{0.001}{0.004}} = 0.0045\,\text{GeV}/c,$$

which, for a 10 cm radius, requires the field:

$$B = \frac{p}{0.3zR} = \frac{0.0045}{0.3(1)(0.1)} = 0.15\,T. \quad (7.3)$$

Problem 7.2 *The mass of a K^+ is 494 MeV/c^2 and that of a π^+ is 140 MeV/c^2. If the rms time resolution of each of two scintillation counters that are 2 m apart is 0.2 nsec, calculate to better than 10% accuracy the momentum at which the system will just be able to resolve a π^+ from a K^+ (by one standard deviation). (Hint: See Eq. (7.10).)*

The task is to find the maximum momentum for which a time resolution $\delta t = 0.2$ ns will just resolve a π meson ($mc^2 = 0.14$ GeV) from a K meson ($mc^2 = 0.49$ GeV) for a 2m flight path. Equation (7.18) is useful only in the approximation that the two masses are nearly equal, which is certainly not the case for these two mesons. You must start with Eq. (7.10), or proceed as follows: assuming a velocity $\beta_\pi = 1$, the time of flight for the π meson is $t = L/c = 7$ ns.

The resolution limit is reached when the uncertainty in the K velocity equals the difference in the two velocities, $\delta\beta_K = \Delta\beta = 1 - \beta_K$. The fractional error in the K velocity is equal to the fractional uncertainty in the time resolution:

$$\frac{\delta\beta_K}{\beta_K} = \frac{\delta t}{t} = 0.03.$$

Using an approximation for large momentum, and solving for p,

$$\frac{\delta\beta_K}{\beta_K} = \frac{1}{\beta_K} - 1 = \sqrt{1 + \frac{m_K^2c^2}{p^2}} - 1 \approx \frac{m_K^2c^2}{2p^2} \quad \text{for } p^2 \gg m^2c^2$$
$$\text{and} \quad p = \frac{m_K}{\sqrt{\frac{2\delta\beta_K}{\beta_K}}} = \frac{0.5}{\sqrt{0.06}} = \frac{0.5}{0.25} = 2.0\,\text{GeV}/c. \quad (7.4)$$

Problem 7.3 *What are the Cherenkov angles for electrons and pions of 1000 MeV/c for a radiator with $n = 1.4$? What will be the ratio of the number of radiated photons for incident electrons and pions?*

The Cherenkov angle is related to the velocity of the particle and index of refraction n of the medium.

$$\cos\theta_c = \frac{1}{\beta n}. \quad (7.5)$$

The Cherenkov angles for electrons and π mesons of 1 GeV/c, passing through radiator of $n = 1.4$, are

$$\text{electron}: \theta_c = \cos^{-1}\left[\frac{1}{(1)1.4}\right] = 44.4^\circ,$$
$$\text{pion}: \theta_c = \cos^{-1}\left[\frac{1}{(0.99)1.4}\right] = 43.8^\circ. \quad (7.6)$$

The number of radiated photons is related to the Cherenkov angle by

$$N \propto \sin^2\theta_c \quad (7.7)$$

so that the ratio of the number of photons emitted by an electron to those for a π meson is

$$\frac{N_e}{N_\pi} = \left[\frac{\sin(44.4^\circ)}{\sin(43.8^\circ)}\right]^2 = 1.02. \quad (7.8)$$

Problem 7.4 *About 10^6 electron-ion pairs are produced by a charged particle traversing a counter. If the typical ionization potential of the medium is $\bar{I} = 30$ eV, in principle, how well can you measure the deposited energy using a Geiger counter, an ionization*