CHAPTER 13
Formulation of the Standard Model

Lecture Notes For
PHYS 415
Introduction to Nuclear and Particle Physics

To Accompany the Text
Introduction to Nuclear and Particle Physics, 2nd Ed.
A. Das and T. Ferbel
World Scientific

Hadrons are Made of Quarks

- By the 1960s there were a host of particles.
- Even the lightest of the baryons showed evidence of substructure.
- From the pattern of the spectrum of hadrons, Gell-Mann and Zweig suggested that such particles consisted of **quarks**, presumed to be fundamental.
- At first quarks were assumed to be merely convenient calculational constructs. However, they have come to be regarded as real physical objects.

Quarks "Discovered"

- Experiments performed at SLAC (Friedman, Kendall and Taylor) in the late 1960s gave the first experimental evidence of the quark model.
- Electron scattering from hydrogen and deuterium targets, pointed to the existence of point-like constituents of the proton and neutron.
- The nucleons are now known to contain quarks (of charge +2/3 *e* and -1/3 *e*) as well as neutral **gluons**, the carriers of the strong force.
- In contrast, the leptons do not exhibit any structure, at the distance scales probed so far.

Isolated Quarks Never Observed

- The quark discovery is of a quite different nature than previous particle discoveries.
- Quarks, and therefore fractional charges, are never observed in isolation.
- They are rather inferred from a large body of data and from underlying symmetry principles.
- The nonexistence of isolated quarks is related to the concept of **confinement** of the strong interaction.

Quarks and Leptons

- There are three families of leptons:

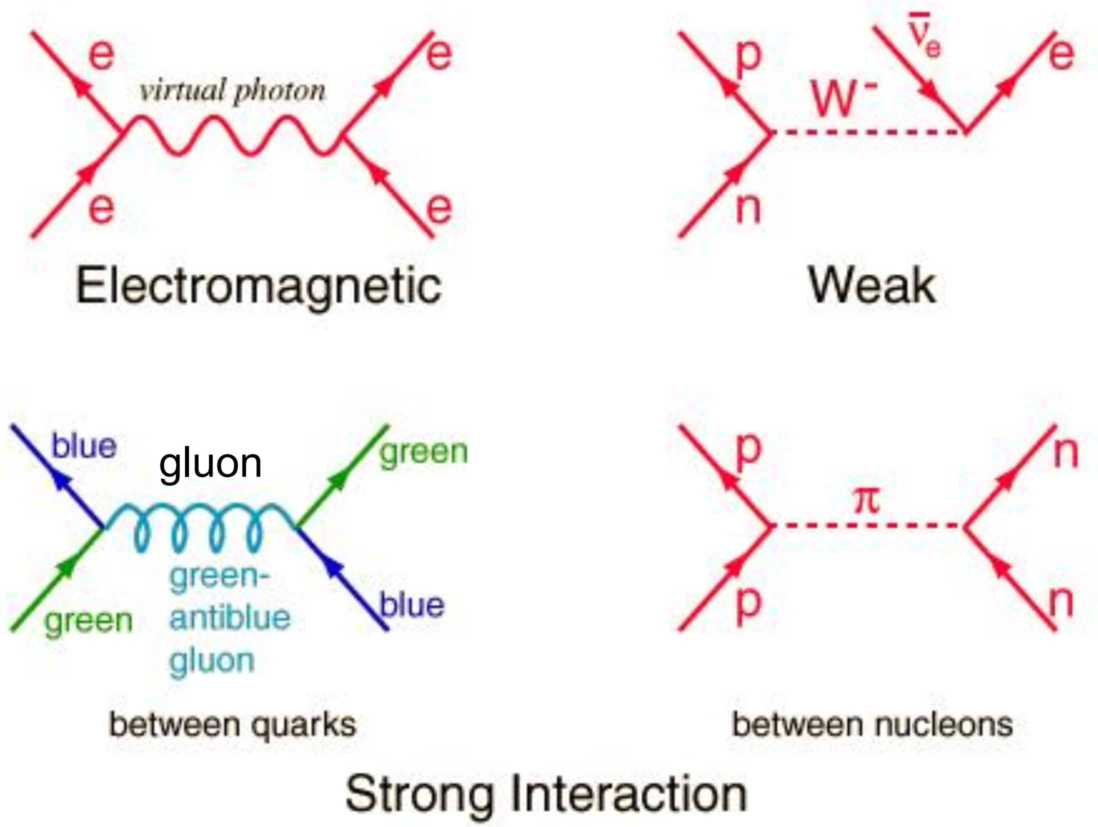
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

  and three families of quarks:

$$\begin{matrix} Q = +\frac{2}{3}e \longrightarrow \\ Q = -\frac{1}{3}e \longrightarrow \end{matrix} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

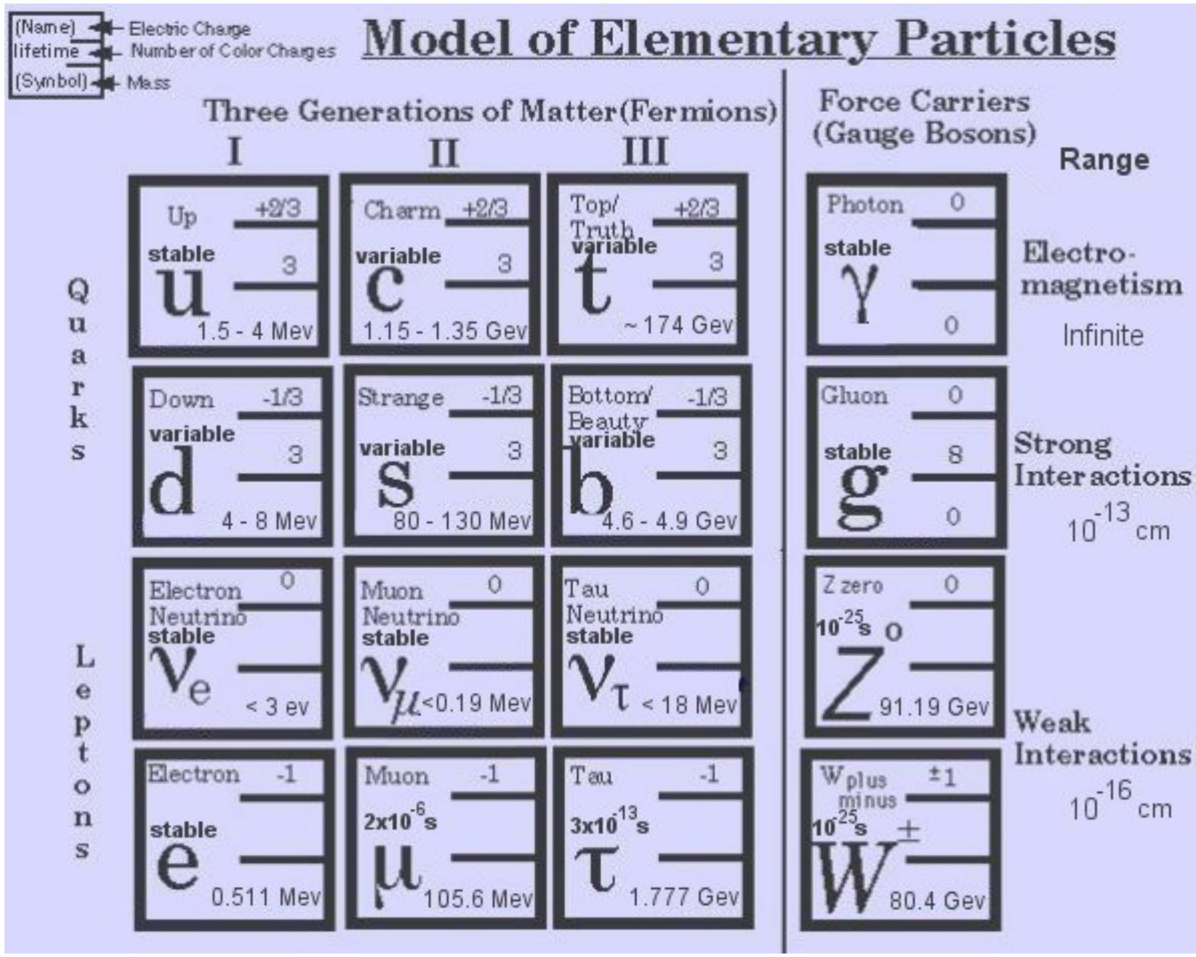
- Quarks also have flavor quantum numbers (strangeness, charm, …).
- In addition to these particles we also have all the corresponding anti-particles as well as the gauge bosons which carry the various forces …

Gauge Bosons



Adapted from http://universe-review.ca/I15-53-electroweak.jpg

Summary of Elementary Particles



From http://universe-review.ca/I15-02-elementary.jpg

Mesons and Baryons

- Quarks are point-like fermions, with spin 1/2.
- Mesons have integer spin ⇒ composed of even number of quarks. All known mesons are composed of a quark-antiquark pair.
- Baryons have half-integer spin ⇒ composed of odd number of quarks. All known baryons are composed of three quarks. Evidence for "Pentaquarks" is highly controversial so far.

Quark Content of the Pions

- The pion belongs to a strong isospin triplet. The quark configuration of the π^- and π^+ follow from charge considerations. Other considerations of symmetry dictate the quark content of the π^0 :

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} = \begin{pmatrix} u\bar{d} \\ \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \\ \bar{u}d \end{pmatrix}$$

- Note, this implies the π^- and π^+ are antiparticles, while the π^0 is its own antiparticle.

Strange Mesons

- The kaons are:

$$K^+ = u\bar{s}$$

$$K^- = \bar{u}s$$

$$K^0 = d\bar{s}$$

$$\overline{K^0} = \bar{d}s$$

- The strangeness assignments we had previously work out, provided we assign strangeness *S* = -1 to the *s*-quark.
- Heavier mesons containing higher mass quarks have also been found …

Charmed Mesons

- The charge-neutral J/ψ meson: $c\bar{c}$
- We also have "open-charm":

charmed:

$$D^+ = c\bar{d} \qquad D^- = \bar{c}d \qquad D^0 = c\bar{u} \qquad \overline{D^0} = \bar{c}u$$

charmed and strange:

$$D_s^+ = c\bar{s} \qquad D_s^- = \bar{c}s$$

- The D^+ meson has charm flavor +1 ⇒ the *c*-quark has charm quantum number +1.
- There are also *bottom* mesons containing a bottom quark or antibottom quark.

Quark Content of Baryons

- Baryons are composed of three quarks:

$$I = \frac{1}{2}, \quad S = 0: \qquad p = uud \qquad n = udd$$

$$I = 0, \quad S = -1: \quad \Lambda^0 = uds$$

$$I = 1, \quad S = -1: \quad \Sigma^+ = uus \quad \Sigma^0 = uds \quad \Sigma^- = dds$$

$$I = \frac{1}{2}, \quad S = -2: \quad \Xi^0 = uss \quad \Xi^- = dss$$

- Baryon # of quarks = 1/3
  - Baryon # of baryons = 1
  - Baryon # of mesons = 0

Need for Color

- Consider Δ^{++} baryon with spin = 3/2, charge = +2*e*:

$$\Delta^{++} = uuu$$

- The Δ^{++} is composed of three identical fermions and so the wave function should be antisymmetric under quark interchange.
- Since the quark spins are all parallel in the ground state (ℓ =0, s = 3/2 $\Rightarrow$ J = 3/2), the wave function is symmetric under quark interchange. This violates the Pauli principle.
- Solution: the quarks carry an additional quantum number, called **color**. The color part of the wavefunction is antisymmetric under interchange.
- This also explains the spin 3/2 Ω^- baryon: $\Omega^- = sss$.

Color

- Each quark carries a color index:

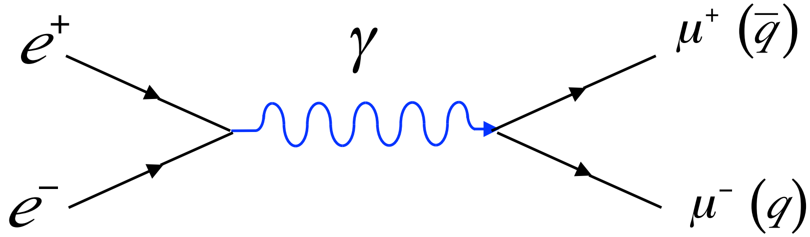
$$\begin{pmatrix} u^a \\ d^a \end{pmatrix}, \quad \begin{pmatrix} c^a \\ s^a \end{pmatrix}, \quad \begin{pmatrix} t^a \\ b^a \end{pmatrix}, \qquad a = \text{red, blue, green}$$

- Hadrons are colorless (or color singlets).
  - Baryons
    - red + green + blue = colorless
    - quark interchange $\Rightarrow \psi_{\text{color}} \rightarrow -\psi_{\text{color}}$
  - Mesons
    - red + anti-red = colorless, …
    - quark interchange $\Rightarrow \psi_{\text{color}} \rightarrow +\psi_{\text{color}}$

Evidence for Three Colors

- Consider electron-positron annihilation:

$$R = \frac{\sigma\left(e^- e^+ \to \text{hadrons}\right)}{\sigma\left(e^- e^+ \to \mu^- \mu^+\right)}$$



- The ratio *R* is proportional to the number of quark colors. The experimental results are consistent with exactly three colors.
- The hadron production cross sections also confirm the fractional nature of the quark charges.

Quark Model for Mesons

- Consider the charge neutral mesons in a non-relativistic quark model. From symmetry arguments we can work out the expected quantum numbers, J^{PC}.
- The wave function can be expressed as:

$$\Psi = \psi_{\text{space}} \psi_{\text{spin}} \psi_{\text{charge}}$$

- Now consider how Ψ transforms under quark interchange (the color part of the wave function has been ignored, since that always has even symmetry for mesons) …

Quark Exchange Properties of Ψ_{meson}

- Define *X* as the quark-antiquark exchange operator. The spatial part of the wave function transforms as:

$$X\psi_{\text{space}} \sim X\,Y_{m}(\theta,\phi) \sim (-1)\;\psi_{\text{space}}$$

- The spin symmetry will depend on whether the quark and antiquark are in a spin 0 (singlet) or spin 1 (triplet) state:

$$\left.\begin{array}{ll} s=0: & X\left[\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right] = -\left[\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right] \\ s=1: & X\left[\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle\right] = +\left[\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle\right] \end{array}\right\} \Rightarrow X\psi_{\text{spin}} = (-1)^{s+1}\psi_{\text{spin}}$$

Pauli Principle Applied to Ψ_{meson}

- Invoke a generalized Pauli principle where the quark and antiquark are regarded as identical fermions corresponding to “spin up” and “spin down” states in charge-space.
- Then the overall symmetry must be:

$$\mathcal{X}\Psi = -\Psi$$

- We can use this to determine the charge parity of Ψ …

Charge Parity of Ψ_{meson}

- Interchanging the quark and antiquark is also equivalent to charge conjugation. So the net effect of interchange is:

$$\mathcal{X}\Psi = \mathcal{X}\psi_{\text{space}}\mathcal{X}\psi_{\text{spin}}\mathcal{X}\psi_{\text{charge}}$$

$$= (-1)\ \psi_{\text{space}}(-1)^{s+1}\psi_{\text{spin}}\,\mathcal{C}\psi_{\text{charge}} = -\Psi$$

$$\Rightarrow \eta_{C} = (-1)^{+s} \quad \text{(if meson is eigenstate of } \mathcal{C}\text{).}$$

Parity and Spin of Ψ_{meson}

- The overall parity is determined by the effect of spatial inversion along with the intrinsic parities of the quarks.
- The intrinsic parity of quark and antiquark are opposite, so:

$$\mathcal{P}\Psi = -(-1)\Psi = (-1)^{+1}\Psi \Rightarrow \eta_P = (-1)^{+1}$$

- The spin of the meson is given by the sum of orbital and intrinsic angular momenta:

$$\vec{J} = \vec{L} + \vec{S}$$

Lowest-Lying Meson States

ℓ	s	j	η_P	η_C	Meson
0	0	0	-	+	π^0, η
0	1	1	-	-	ρ^0, ω, ϕ
1	0	1	+	-	$b_1^0(1235)$
1	1	0	+	+	$a_0(1980)$, $f_0(975)$
1	1	1	+	+	$a_1^0(1260)$, $f_1(1285)$
1	1	2	+	+	$a_2^0(1320)$, $f_2(1270)$

Valence and Sea Quarks

- The quarks that characterize the quantum numbers of mesons and baryons are referred to as **valence quarks**.
- In addition, hadrons contain a sea of quark-antiquark pairs, called **sea quarks**, as well as gluons.
- Some evidence exists for other kinds of hadrons:

$$q\bar{q}q\bar{q}, \quad \text{pentaquark } (qqqq\bar{q}),$$

$$\text{hybrid mesons } (q\bar{q}g) \text{ and glueballs } (gg)$$

Weak Isospin

- The concept of strong isospin which applies to hadrons can be extended to include the leptons.
- This isospin is related to the weak interactions of leptons and quarks.
- The so-called weak isospin, like strong isospin, is described by the symmetry group SU(2).
- Weak isospin is only a symmetry when the electromagnetic interactions are ignored.

Weak Isospin Symmetry

- In terms of the weak interaction the "up" and "down" members of each pair are indistinguishable:

$$\underbrace{\begin{pmatrix}\nu_e\\e^-\end{pmatrix},\begin{pmatrix}\nu_\mu\\\mu^-\end{pmatrix},\begin{pmatrix}\nu_\tau\\\tau^-\end{pmatrix}}_{\text{leptons}}\quad\text{and}\quad\underbrace{\begin{pmatrix}u\\d\end{pmatrix},\begin{pmatrix}c\\s\end{pmatrix},\begin{pmatrix}t\\b\end{pmatrix}}_{\text{quarks}}$$

- We can extend the Gell-Mann Nishijima relation to the case of weak isospin, by defining a corresponding **weak hypercharge**:

$$Q = I_3 + \frac{Y}{2} \Rightarrow Y = 2(Q - I_3)$$

weak hypercharge — weak isospin projection

Weak Hypercharge

- Using this relation, we obtain:

$$\begin{cases} Y(\nu) = 2\left(0 - \frac{1}{2}\right) = -1 \\ Y(e^-) = 2\left(-1 + \frac{1}{2}\right) = -1 \\ Y(u) = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3} \\ Y(d) = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{3}, \ \ldots \end{cases}$$

- The SU(2) symmetry applies only to the left-handed leptons. Since there are no right-handed neutrinos, the right-handed leptons are singlets with *I* = 0.
- In the case of strong interactions, the quarks obey a similar symmetry, but since they come in three colors, the symmetry group is SU(3).

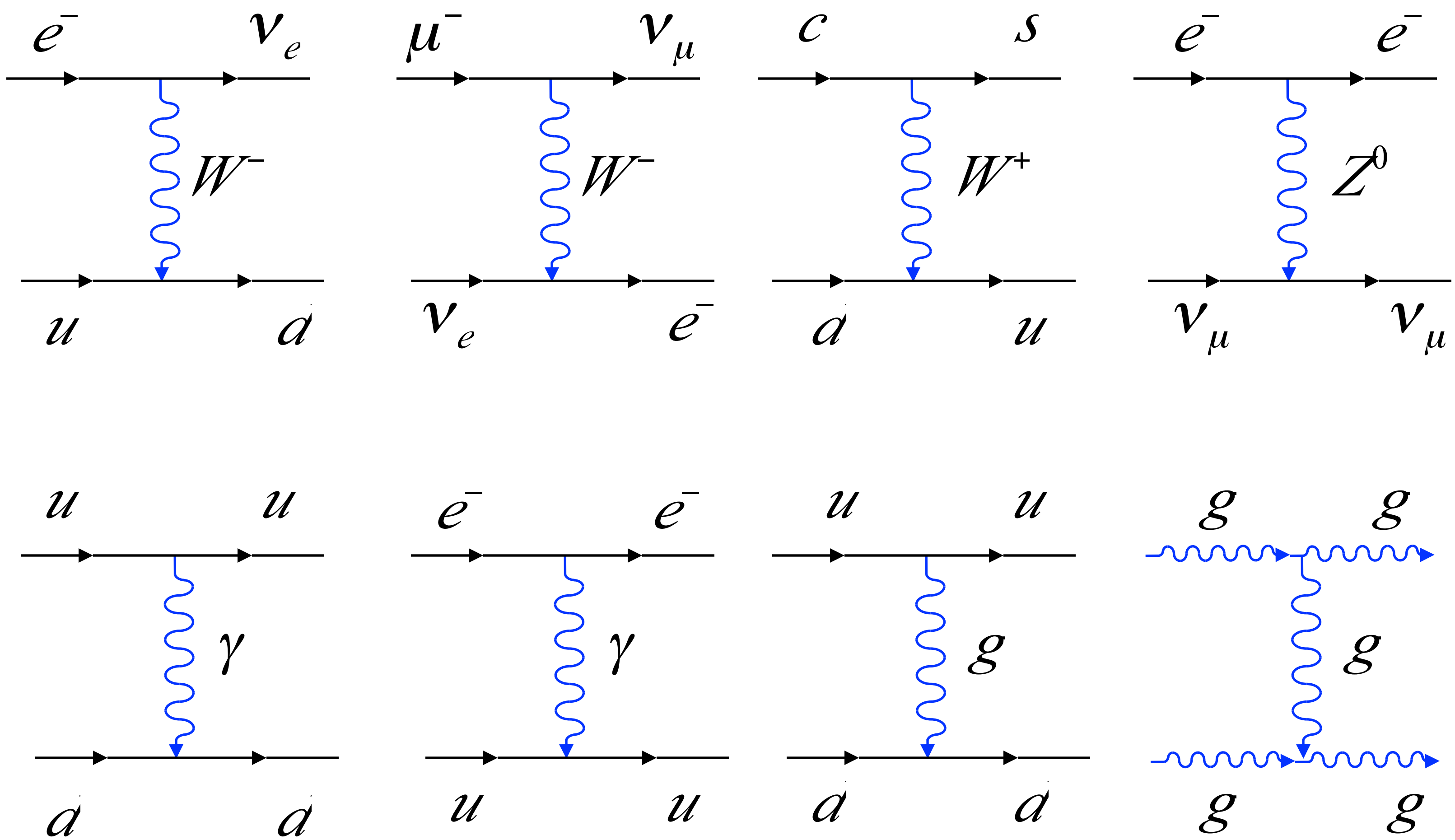
Gauge Bosons

- The various forces appear to arise from local symmetries:

$$\underbrace{\underbrace{U_Y(1)}_{\substack{\text{weak}\\\text{hypercharge}}}\ ,\ \underbrace{SU_L(2)}_{\substack{\text{weak}\\\text{isospin}}}}_{\text{related to EM } U_Q(1) \text{ symmetry}}\ ,\ \underbrace{SU_{\text{color}}(3)}_{\substack{\text{strong}\\\text{interactions}}}$$

- Local symmetry ⇒ invariance ⇒ gauge potential
- Quantization of potential ⇒ gauge particles (force carriers), all of spin 1:
  - EM: 1 gauge boson: photon
  - Weak: 3 gauge bosons: W^+, W^-, Z^0
  - Strong: 8 gauge bosons: gluons
- Whereas the photon is electrically neutral, gluons carry color charge and therefore self-interact. This makes $SU_{color}(3)$ non-Abelian and gives rise to confinement.

Interactions Mediated by Gauge Bosons



Dynamics of the Gauge Particles

- Consider, first, Maxwell's equations in vacuum:

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

- We can write the fields in terms of potentials:

$$\vec{E} = \vec{\nabla} \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Gauge Transformation

- The fields and hence Maxwell's equations are invariant under the gauge transformation:

$$\delta\phi = -\frac{1}{c}\frac{\partial\alpha(\vec{r},t)}{\partial t}$$

$$\delta\vec{A} = \vec{\nabla}\alpha(\vec{r},t)$$

- This invariance also leads to solutions of Maxwell's equations as transverse waves with propagation speed *c*:

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0$$

Gauge Particles

- Quantization of the EM fields leads to massless particles (photons) reflecting the long-range character of the Coulomb force.
- Now consider the equation for a traveling wave for a particle of mass *m*:

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right)\vec{E} = 0$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right)\vec{B} = 0$$

Gauge Particles, cont’d.

- These equations follow from Maxwell-like equations for massive vector fields (J = 1):

$$\vec{\nabla} \cdot \vec{E} = \frac{m^2 c^2}{\hbar^2} \phi$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \frac{m^2 c^2}{\hbar^2} \vec{A}$$

- These equations depend explicitly on the potentials and so are only gauge invariant when $m = 0 \Rightarrow$ gauge invariance only holds for massless gauge bosons.

Short-Range Forces

- Even though the weak and strong interactions are of short range, they are still based on gauge principles.
- For the weak force, the short-range character arises from the mechanism of **spontaneous symmetry breaking** resulting in massive force carriers.
- For the strong force, the short-range character arises from gluon self-interaction. The gluon, like the photon, is massless.

Symmetry Breaking: Ferromagnets

- Consider a ferromagnet described by a nearest neighbor interaction: $$H = -\kappa \sum_i \vec{s}_i \cdot \vec{s}_{i+1}$$
- If all spins are rotated through the same angle, *H* is invariant.
- However, the ground state has all spins parallel. The orientation of spins gives a preferred direction, picked out at random.
- The ground state does not share the symmetry of *H*. This spontaneously broken symmetry arises from effective long range interactions, corresponding to zero mass particles in the quantum theory.
- The symmetry of the ground state can be restored by raising the temperature of the ferromagnet above the Curie temperature.

Another Example: Mexican Hat Potential

- Consider a potential, symmetric about the *z*-axis, giving a Hamiltonian:

$$H = T + V = \frac{1}{2m}\left(p_x^2 + p_y^2\right) - \frac{1}{2}m\omega^2\left(x^2 + y^2\right) + \frac{\lambda}{4}\left(x^2 + y^2\right)^2, \quad \lambda > 0$$

From http://upload.wikimedia.org/wikipedia/en/3/3e/Spontaneous_symmetry_breaking.jpg

Minimum Energy Solutions

- Minimizing the energy amounts to finding extrema of *V*, giving:

$$x_{\min} = y_{\min} = 0, \quad \text{or} \quad x_{\min}^2 + y_{\min}^2 = \frac{m\omega^2}{\lambda}$$

- The first condition actually gives an unstable maximum. The second gives the minimum energy and corresponds to a circle centered on the *z*-axis.
- Let's pick a particular solution:

$$y_{\min} = 0 \quad \text{and} \quad x_{\min} = \sqrt{\frac{m\omega^2}{\lambda}}$$

Stability of Solution

- Expand the potential about the coordinates of the minimum:

$$V\left(x_{\min}+\delta_x, y_{\min}+\delta_y\right) = -\frac{1}{2}m\omega^2\left[\left(x_{\min}+\delta_x\right)^2+\delta_y{}^2\right]+\frac{\lambda}{4}\left[\left(x_{\min}+\delta_x\right)^2+\delta_y{}^2\right]^2$$

$$\xrightarrow{\text{2nd order in } \delta_x, \delta_y} -\frac{m^2\omega^4}{4\lambda}+m\omega^2\delta_x{}^2 + \text{ higher orders}$$

- Frequencies of small oscillations are therefore:

$$\omega_x = \sqrt{2}\omega \quad \text{and} \quad \omega_y = 0$$

- A similar result follows for any choice of coordinates along the circle defining the minimum of *V*. Motion away from the *z*-axis requires energy whereas motion along the valley requires no energy (zero frequency mode).

Symmetry Breaking in Quantum Field Theory

- In QFT, an analogous result follows:
  - The zero frequency mode gives rise to massless particles: Nambu-Goldstone bosons.
  - The non-zero frequency mode gives rise to massive particles.
- When the broken symmetry is local, rather than global, the Nambu-Goldstone bosons become the longitudinal modes of the gauge bosons.
- The resultant gauge bosons develop mass and the corresponding fields lose their purely transverse character.

Higgs Mechanism

- The example given is analogous to the broken symmetry in the case of the weak interaction, the Higgs mechanism.
- The weak gauge bosons develop mass and the force becomes short-ranged.
- The massive partner of the Goldstone bosons is called the **Higgs** particle (discovered at LHC in 2012).
- Weak hypercharge and isospin are spontaneously broken, but the combination leading to $U_Q(1)$ is not.
  - The photon remains massless.
  - The weak bosons become massive.

Unification of Forces

- The ferromagnet, which has a preferred orientation of ground state spins, becomes rotationally invariant at high temperatures.
- Beyond some energy scale weak-isospin symmetry is restored and the weak bosons become massless just like the photon.
- Thus, at sufficiently high energy the forces may be indistinguishable. This is consistent with the earlier statement that the strengths of the weak and electromagnetic forces may become comparable at high energy.

Quantum Chromodynamics (QCD)

- The theory of strong interactions, Quantum Chromodynamics, is governed by the non-Abelian color symmetry group $SU(3)$.
- The gauge bosons of the color symmetry are the gluons which, like the photon, are massless.
- Unlike photons, gluons interact with themselves (they carry color charge).
- Also, unlike how charges add in QED, a color singlet object can be obtained in two ways:
  - red + anti-red = color neutral (meson)
  - red + blue + green = color neutral (baryon)

Asymptotic Freedom

- The field of electric charges is reduced in a dielectric due to polarization of the medium (pairs of oppositely charged particles, dipoles, reduce the electric field).
- Even in vacuum, quantum fluctuations give rise to a similar screening effect. The result is a slight increase of the fine structure constant, α, with increasing momentum transfer (i.e. at short range).
- In contrast, due to the way in which color neutral objects may be produced, the strong interaction is *anti-screened*: it becomes weaker at short distance (high momentum transfers).
- This property gives rise to **asymptotic freedom** of QCD. Quarks behave as essentially free particles at high energy $\Rightarrow$ parton model. This fact is exploited in perturbative QCD; such calculations and experiment are in excellent agreement.

Confinement

- At low energies, the color couplings increase, leading to the existence of QCD bound states: hadrons.
- Quarks account for roughly half the momentum of hadrons, the other half is carried by the gluons.
- Quarks are confined to hadrons. This result can be understood qualitatively by considering a simple form for the potential between quarks …

Quark Potential

- Assuming a potential, $V(r) \propto kr$, as a quark and antiquark are pulled apart, the energy increases linearly.
- This effect can be likened to a string stretched between the quarks. The gluon self-interaction causes the color field to be essentially localized to a tube: **flux-tube**.
- As the energy in the tube grows, it becomes energetically favorable to produce a quark-antiquark pair from the vacuum. Each member of the pair associates with one of the original quarks, producing two colorless particles.

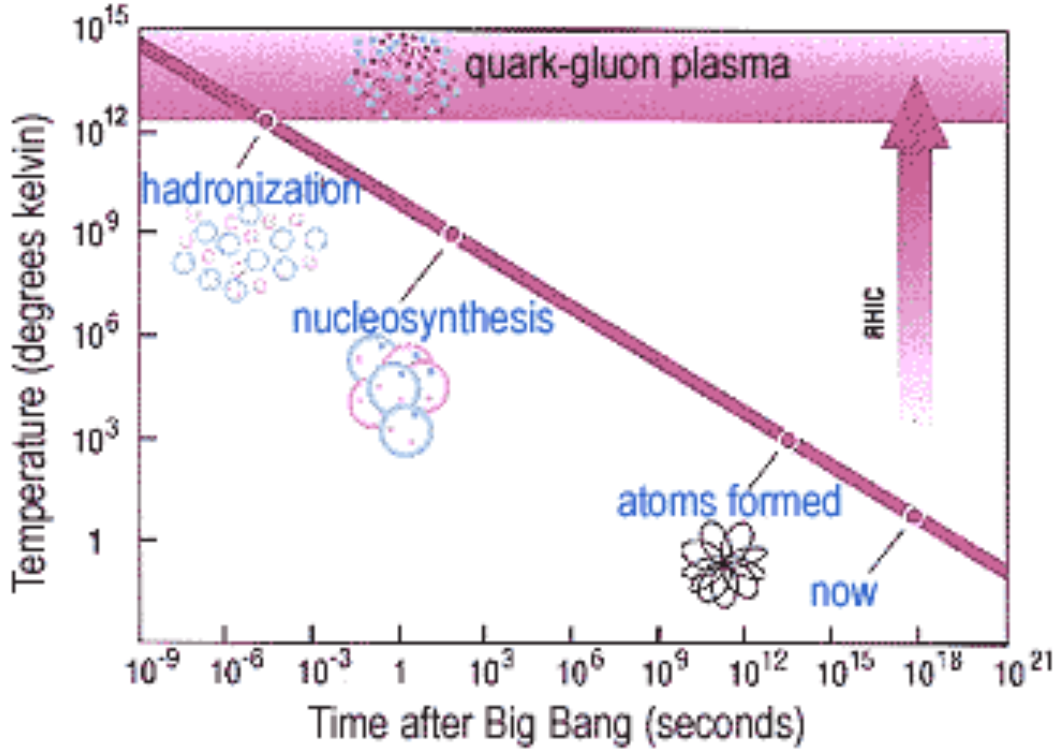
Residual Interaction

- The short range range nature of the force between hadrons arises from a "residual" color interaction.
- This is analogous to the Van der Waals force for neutral molecules. This force, arising from the field of an electric dipole, is relatively weak and short-ranged, compared to the Coulomb force between isolated charges.
- For the strong force, the residual interaction can be described by the exchange of mesons, rather than gluons.

Quark-Gluon Plasma

- Though quarks are confined within hadrons, at sufficiently high temperatures, a new state of matter is hypothesized: the **quark-gluon plasma** (QGP).
- The QGP is a phase of matter in which quarks and gluons are essentially free.
- The conditions for the QGP would have existed in the early universe and perhaps inside dense neutron stars …

Quark-Gluon Plasma, cont’d.



From http://www.bnl.gov/RHIC/images/bigbang_graph-w.gif

- Recent experiments at RHIC and LHC using Au + Au collisions claim strong evidence in support of a liquid phase QGP.