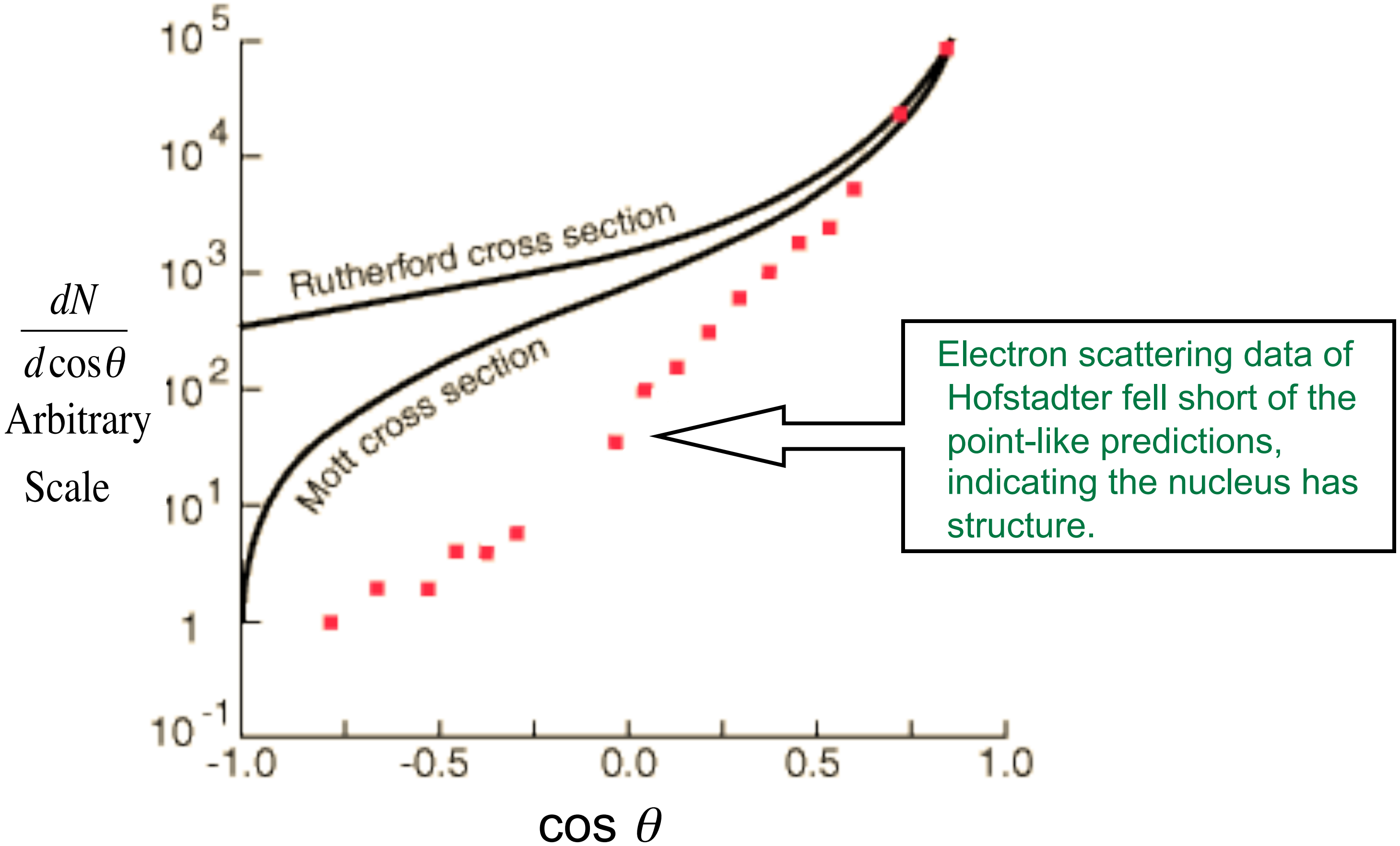
CHAPTER 2
Nuclear Phenomenology

Lecture Notes For
PHYS 415
Introduction to Nuclear and Particle Physics

To Accompany the Text
Introduction to Nuclear and Particle Physics, 2nd Ed.
A. Das and T. Ferbel
World Scientific

The Nucleus is not Point-like



R. Hofstadter, *et al*., Phys. Rev. **92**, 978 (1953).

Figure adapted from http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/elescat.html

Deviations from Rutherford

- For incident particles of higher energy and/or low Z nuclei, deviations from Rutherford prediction were observed.
- High energy $\Rightarrow$ distance of closest approach is small. Low $Z \Rightarrow$ same, since Coulomb force is weaker.
- The nucleus itself was being probed.
- Nucleus is not point-like and force is not Coulomb force.

Properties of Nuclei

- Nuclei consist of protons and neutrons. (Heisenberg uncertainty principle: suggests electrons cannot exist inside nucleus.)
- Notation: N = # neutrons
  Z = # protons
  $A = N + Z$
  Nucleus X: ${}^{A}X^{Z}$
- Isotopes: ${}^{A}X^{Z}$ and ${}^{A'}X^{Z}$
- Isobars: ${}^{A}X^{Z}$ and ${}^{A}Y^{Z'}$
- Isotones: same number of neutrons

Nuclear Masses

- To first order: $M(A,Z) = Zm_p + (A-Z)m_n$
  - m_p = proton mass $\approx$ 938.27 MeV/c^2
  - m_n = neutron mass $\approx$ 939.56 MeV/c^2
- If this were true, then the nucleus would be unstable and could simply break apart into its constituents.
- The nucleus is a **bound** system and so its mass is less than this simple estimate:

$$\Delta M(A,Z) = M(A,Z) - Zm_p - (A-Z)m_n = \text{B.E.}/c^2 < 0$$

Binding Energy per Nucleon

$$\frac{B}{A} = \frac{-\text{B.E.}}{A} = \frac{-\Delta M(A,Z)c^2}{A} = \frac{Zm_p + (A-Z)m_n - M(A,Z)}{A}c^2$$

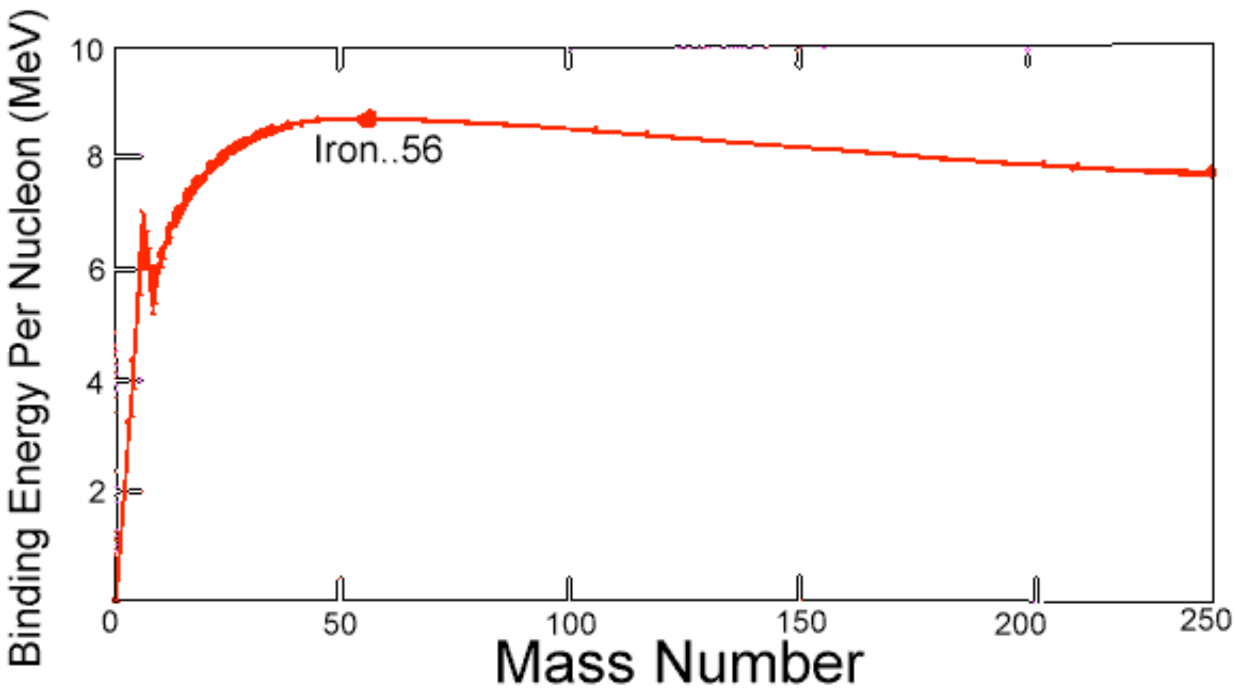


Figure from http://library.thinkquest.org/3471/mass_binding_body.html

Implications

- ^{56}Fe is the most stable nucleus.
  - Higher mass nuclei will release energy to make two smaller nuclei: **Fission**
  - Lower mass nuclei will release energy when combined into a higher mass one: **Fusion**
- Rule of thumb: B.E./nucleon ~ 8 MeV
- If we supply 8 MeV, all to one nucleon, then we can free it from the nucleus.

Implications, cont'd.

- If we give 8 MeV to a single nucleon:

$$\lambda\!\!\!^{-} = \frac{\hbar}{p} = \frac{\hbar c}{\sqrt{2mc^2T}} \approx 1.6 \text{ fm}$$

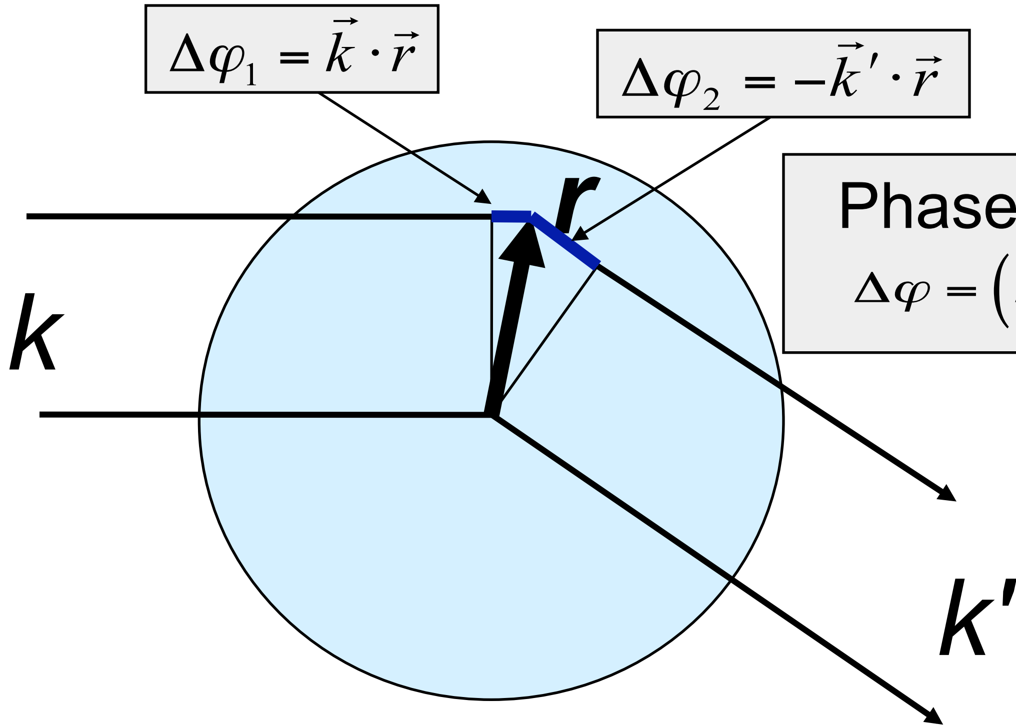
- This is a typical nuclear dimension
  - Nuclei can absorb or emit nucleons of this energy
- For an 8 MeV electron: $\lambda\!\!\!^{-} = \frac{\hbar}{p} = \frac{\hbar c}{T} \approx 25 \text{ fm}$
  - 8 MeV electrons will not fit!
  - 120 MeV electrons would fit, but are not consistent with typical binding energies.

Nuclear Sizes

- Cannot calculate without knowing the nuclear force.
- Can use low-energy α backscattering (distance of closest approach is a minimum) to estimate the size: get upper limits of few 10's of fm. Not too precise!
- Can use high energy electron scattering
  - Not sensitive to nuclear force. EM interaction is known and can be used to determine distribution of charge and magnetism in the nucleus.
  - Can penetrate deeply into the nucleus.

$\Rightarrow$ Determine nuclear **form factors**

Form Factor



Phase difference:

$$\Delta\varphi = \left(\vec{k}-\vec{k}'\right)\cdot\vec{r} = \frac{\vec{q}}{\hbar}\cdot\vec{r}$$

Amplitude at q: $F(\vec{q}) = \int d^3r\rho(\vec{r})e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}}$

Form Factor and Charge Radius

- The *charge form factor* is $F(\vec{q}) = \int d^3 r \rho(\vec{r}) e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}}$
- If the charge density is spherically symmetric, we can integrate over angles explicitly:

$$F(q) = \frac{4\pi\hbar}{q}\int_0^\infty dr \rho(r) r \sin\frac{qr}{\hbar}$$

$$\xrightarrow{qr<<\hbar} \frac{4\pi\hbar}{q}\int_0^\infty dr \rho(r) r \left(\frac{qr}{\hbar} - \frac{1}{6}\left(\frac{qr}{\hbar}\right)^3 + \cdots \right)$$

$$= \int_0^\infty dr \rho(r) 4\pi r^2 - \frac{1}{6}\left(\frac{q}{\hbar}\right)^2 \int_0^\infty dr\, r^2 \rho(r) 4\pi r^2 + \cdots$$

$$= 1 - \frac{1}{6}\left(\frac{q}{\hbar}\right)^2 \left\langle r^2 \right\rangle + \cdots$$

Charge Radius

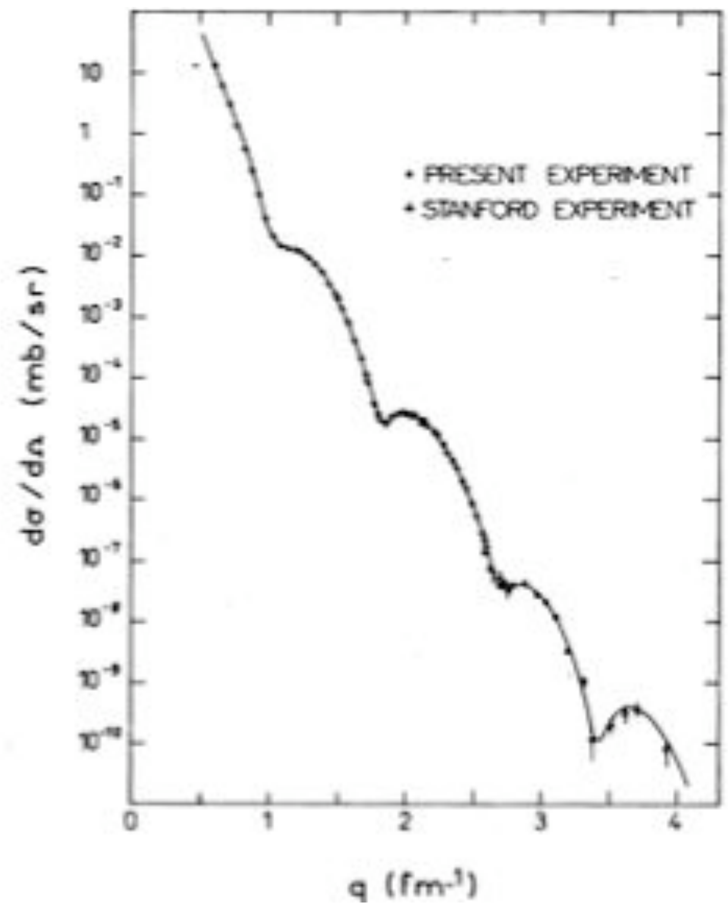
- The slope of the form factor at low q^2 gives the rms charge radius:

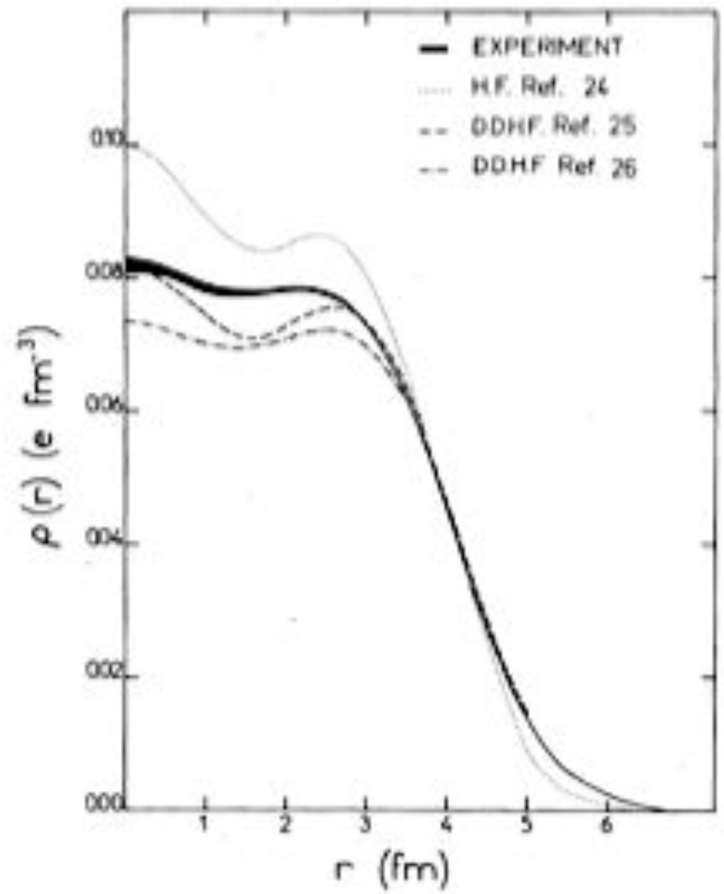
$$\left\langle r^2 \right\rangle = -6\hbar^2 \left. \frac{dF(q)}{dq^2} \right|_{q^2 \to 0}$$

- Further, the charge density can be determined from the form factor via the inverse Fourier transform:

$$\rho(r) = \frac{1}{2\pi^2 \hbar^2 r} \int_0^\infty F(q) \sin\frac{qr}{\hbar}\; q\, dq$$

Example: Charge Density of ^{58}Ni





Elastic electron scattering: I. Sick *et al*., Phys. Rev. Lett. **35**, 910 (1975).

Electron Scattering Cross Section

- Neville Mott considered effect of electron spin in scattering from a nucleus. The Rutherford formula has to be modified:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$$

- This gives the scattering of (point-like) spin-1/2 electrons from a spinless, infinitely massive point-like nucleus.
- To include nuclear size, we insert the form factor:

$$\frac{d\sigma}{dq^2} = \left|F(\vec{q})\right|^2\left(\frac{d\sigma}{dq^2}\right)_{Mott}$$

- We can also include (i.e. determine) the magnetic form factor as well as a factor accounting for the finite nuclear mass (i.e. nuclear recoil).

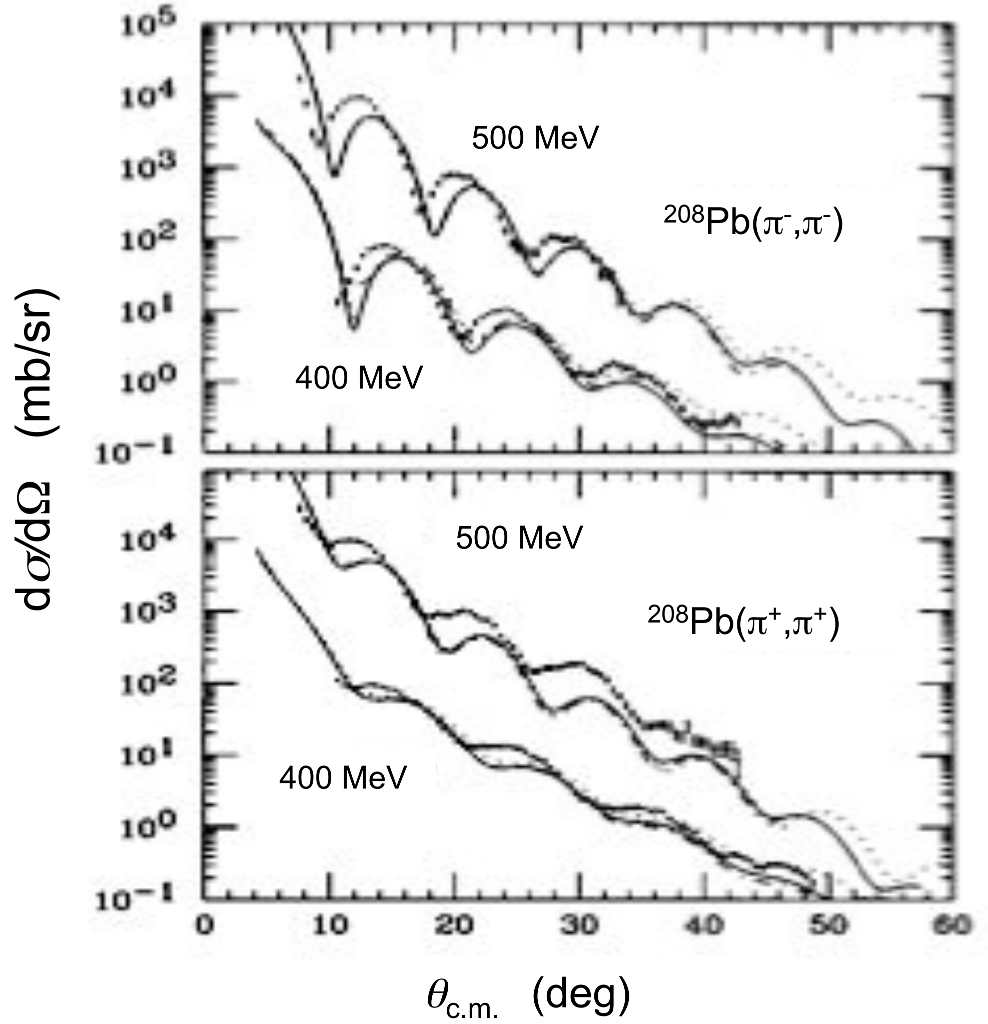
Nuclear Sizes

- The above can be used to determine the nuclear form factor:

$$\frac{\left(\frac{d\sigma}{dq^2}\right)_{measured}}{\left(\frac{d\sigma}{dq^2}\right)_{Mott}} = \left|F(\vec{q})\right|^2$$

- We can also scatter strongly interacting particles such as pions. The nuclei effectively absorb pions out of the beam. The resulting diffraction pattern (similar to diffraction of light by a disk) can be used to determine the size of the nucleus.

Pion Elastic Scattering from Lead



G. Kahrimanis *et al.*, Phys. Rev. C **55**, 2533 (1997).

Nuclear Sizes, an Empirical Formula

- A wide body of such experiments indicates nuclear sizes follow a very simple empirical formula:

  $$R = r_0 A^{1/3} \approx (1.2\ \text{fm})\ A^{1/3}$$

  where A is the mass number.
- The volume is proportional to A and the density is independent of A. This suggests the nucleus can be approximated for certain purposes as an incompressible liquid droplet.

Nuclear Spins and Dipole Moments

- Like the electron, the proton and neutron are both spin-1/2: $S_z = \pm \frac{\hbar}{2}$ and $\left|\vec{S}\right| = \sqrt{s(s+1)}\,\hbar \xrightarrow{s=1/2} \frac{\sqrt{3}}{2}\hbar$
- Nuclear spin is the sum of nucleon spins and orbital angular momenta: $\vec{J} = \sum_{nucleons}\left[\vec{L}_i + \vec{S}_i\right]$
- For charged particles, the spin gives rise to a magnetic moment:

$$\vec{\mu} = g\frac{e}{2mc}\vec{S} \quad \stackrel{spin\ 1/2}{\longrightarrow} \quad \mu = \frac{e\hbar}{2mc}$$

where g = "gyromagnetic ratio" = 2, for a point-like Dirac particle

g-factors

- For the electron
  - $g_e - 2 \approx 2.3\times10^{-3}$:
  - $g_e = 2.0023193043718 \pm 0.0000000000075$
  - The value agrees with the QED prediction which is of comparable accuracy!
- For the proton
  - $g_p/2 \approx +2.79$
  - Strong indication of internal structure.
- For the neutron
  - $g_n/2 \approx -1.91$
  - For a neutral object, expect $g = 0 \Rightarrow$ neutron has an extended charge distribution.

Bohr Magneton and Nuclear Magneton

- From before (for spin-1/2): $\mu = \frac{g}{2}\frac{e\hbar}{2mc}$

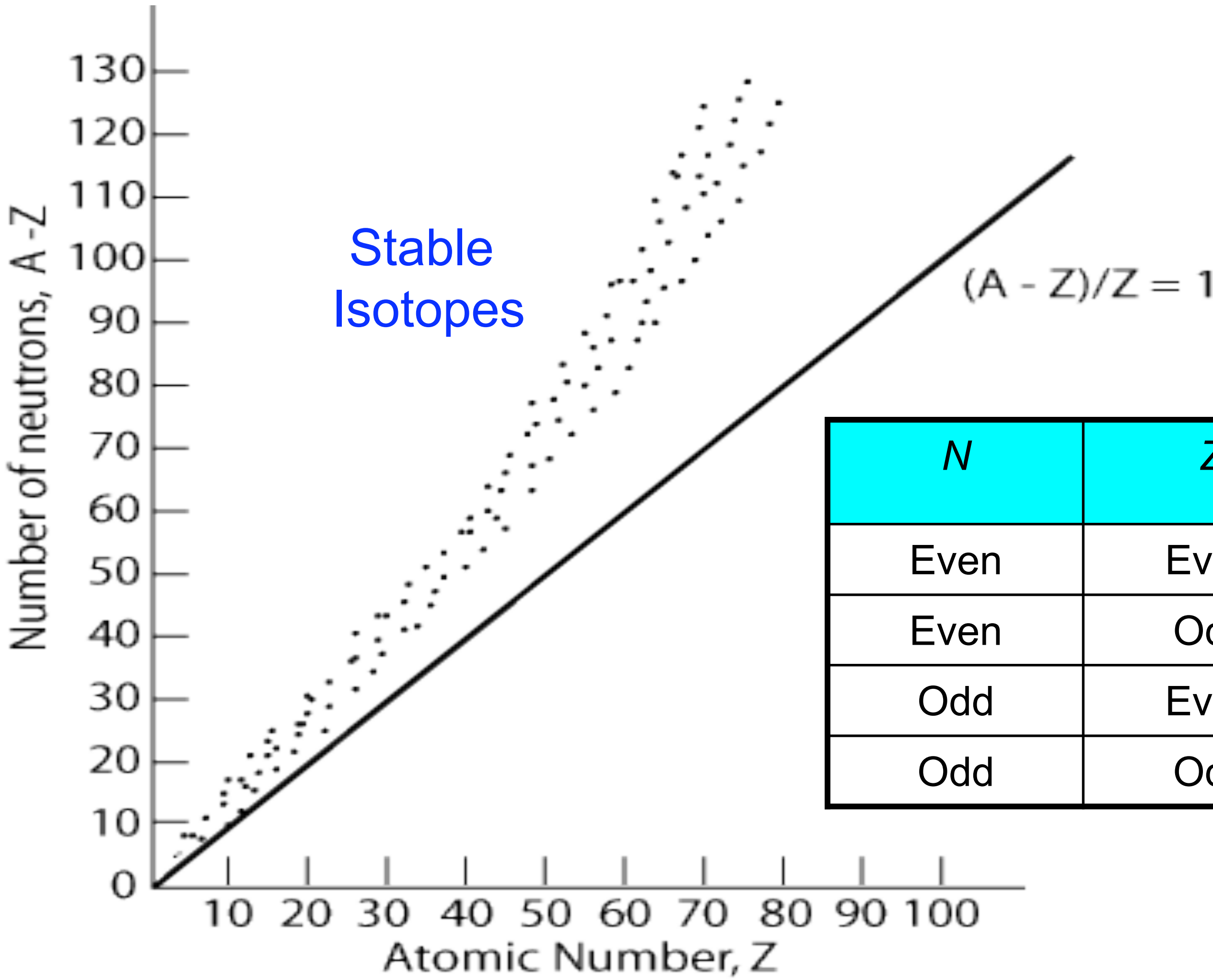
- Bohr magneton: $\mu_B \equiv \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11}\ \text{MeV/T}$
- Nuclear magneton: $\mu_N \equiv \frac{e\hbar}{2m_p c}$

- Due to the mass dependence, the Bohr magneton is ~ 2000 times larger than the nuclear magneton.

Nuclear Spin and Magnetic Moment

- General observations about spin
  - For *A* even: integral spin
  - For *A* odd: half-integral spin
  - For *N* and *Z* even: spin = 0, always
  - Even large nuclei have small ground state spins
  - Suggests that spins are strongly paired in nuclei
- Magnetic moments
  - All measured values lie between $-3\mu_N$ and $10\mu_N$
  - Additional evidence for strong pairing
  - Difficult to accommodate electrons within the nucleus, given the much larger electron magnetic moment

Nuclear Stability



N	*Z*	# of Stable Nuclei
Even	Even	156
Even	Odd	48
Odd	Even	50
Odd	Odd	5

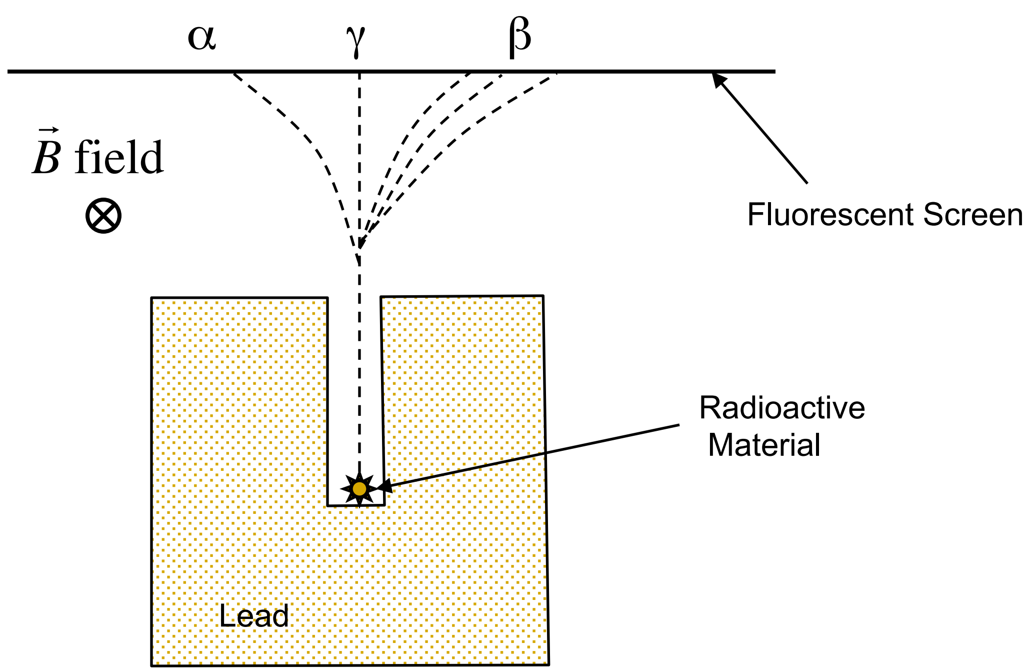
From http://www.algebralab.org

Nuclear Stability, cont'd.

- For light nuclei: $N \approx Z$
- For heavier nuclei: $N \approx 1.7\ Z$
  - Neutron excess reflects smaller overall Coulomb repulsion and therefore higher stability
- Even # of protons/neutrons is favored
  - Further evidence of strong pairing, i.e. pairing of nucleons leads to nuclear stability

Nuclear Instability: Radioactivity

- Discovered in uranium salts by Henri Becquerel (1896).
- Three basic types: α, β and γ



Radiation

- Various sheets of materials could be used to study the range of each type of particle and therefore establish the degree to which it ionizes matter.
- A superimposed electric field could be arranged to establish the charge-to-mass ratio of each particle.
- Results:
  - α = *helium nucleus*, small range, heavily ionizing
  - β = *electron*, longer range, less heavily ionizing
  - γ = *photon*, longest range, least ionizing

Nature of the Nuclear Force

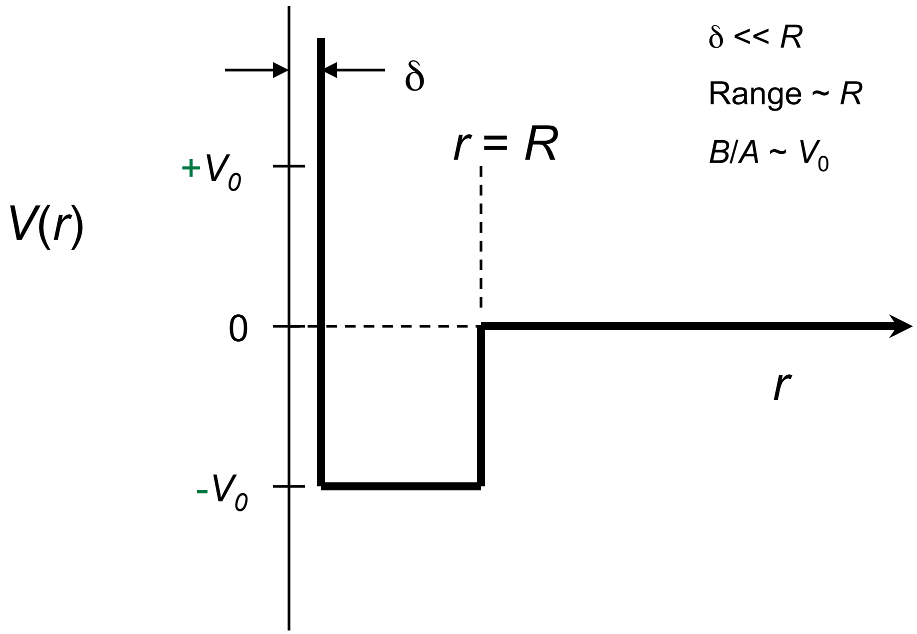
- A new type of force is needed to bind nuclei
  - Gravity is too weak to bind
  - EM cannot bind the deuteron and leads to instability for other nuclei (repulsive force between like charged protons)
- Range of the nuclear (*strong*) force
  - Atomic structure is well described by just the EM force $\Rightarrow$ range of nuclear force ~ size of nucleus
  - Other evidence for short range: saturation of nuclear force …

Saturation of Nuclear Force

- For a long-range force, such as EM, every particle can interact with all others.
- This gives # pairwise interactions = $A(A\text{-}1)/2$
- Binding energy: $B \propto A(A\text{-}1) \Rightarrow B/A \propto A$, for large A
  - We would get tighter binding for larger systems.
- But for nuclei: $B/A \approx$ constant
  - Nucleons only interact with a few nearest neighbors.
  - Adding nucleons does not increase average binding energy, but just increases the nuclear size (i.e. density is nearly constant).
  - This is further evidence of short-ranged nature of nuclear force.

Nuclear Force

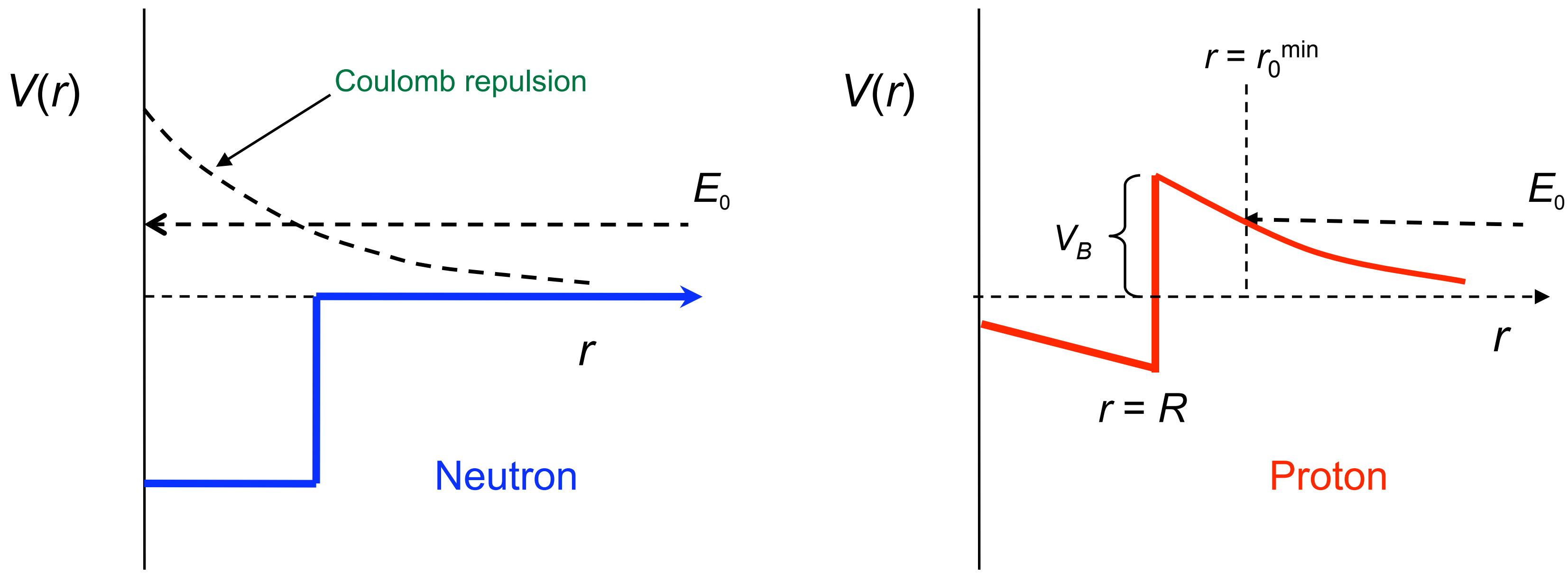
- Nuclear force is
  - Short-ranged
  - Attractive at “long” distances
  - Repulsive at very short distance: *repulsive core*



Crude representation of the nuclear force

Inclusion of Coulomb force

- Neutrons experience no Coulomb force and so even relatively low energy neutrons can penetrate the nucleus.
- Protons of comparable energy will experience an effective (Coulomb) barrier of height V_B.



Repulsive core neglected here

Nuclear Bound States

- The nucleus is a bound system and so exhibits discrete energy levels (bound states).
- These states can be probed in various scattering experiments and the energies can be determined by measuring the particle energy loss and/or various emitted particles from subsequent decay to the ground state.
- In fact, nuclei, just like atoms, are well-described by a shell structure (nuclear shell model).

Charge Independence of Nuclear Force

- The proton-proton, proton-neutron and neutron-neutron forces are the same, once we correct for Coulomb effects.

  $\Rightarrow$ The nuclear force is *charge independent*.
- This is called **isospin symmetry**
  - The proton and neutron can be regarded as two different states of a **nucleon** (analogous to the spin “up” and “down” states of a spin-1/2 particle).
  - In the absence of Coulomb forces, the proton and neutron would be indistinguishable.

Yukawa Potential

- EM force is mediated by the exchange of a (massless) photon giving an infinite range potential:

$$V(r) \propto \frac{1}{r}$$

- Hideki Yukawa (1934) showed that the corresponding potential for a massive (mass = *m*) exchange particle is:

$$V(r) \propto \frac{\exp\left(-\frac{mc}{\hbar} r\right)}{r}$$

Yukawa and Range of the Nuclear Force

- The range of the force varies inversely as the mass of the exchanged particle. This is consistent with the Heisenberg uncertainty principle:
  - The (virtual) particle's energy must be created and therefore is short-lived.
  - A short-lived particle cannot propagate very far.
- The range is related to the (reduced) Compton wavelength:

$$\lambda\!\!\!^{-} = \frac{\hbar}{mc}$$

- Conversely, the mass of the exchanged particle can be deduced from the range of the force:

$$mc^2 = \frac{\hbar c}{\lambda\!\!\!^{-}} \approx \frac{197 \text{ MeV - fm}}{1.2 \text{ fm}} \approx 164 \text{ MeV}$$

The Pion as the Exchange Particle

- This estimate was crude, but the mass is close to that of the pion:
  - $m_{\pi^+} = m_{\pi^-} = 139.6$ MeV/c^2
  - $m_{\pi^0} = 135.0$ MeV/c^2
- The *one-pion exchange* assumption gives a reasonable description of the nuclear force, especially the long-range part.
- However, other, more massive, mesons can also be exchanged. These mainly affect the short-distance character of the nuclear force.