CHAPTER 4
Nuclear Radiation

Lecture Notes For
PHYS 415
Introduction to Nuclear and Particle Physics

To Accompany the Text
Introduction to Nuclear and Particle Physics, 2nd Ed.
A. Das and T. Ferbel
World Scientific

Nuclear Decays

- Three principal decay modes:
  - Alpha decay: $^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}\mathrm{He}^{2}$
  - Beta decay: $^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}$

    $^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu$

    $^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu$
  - Gamma decay: $^{A}X^{*Z} \rightarrow {}^{A}X^{Z} + \gamma$
- Nucleus can also decay via fission into various daughter nuclei (not necessarily α).

Alpha Decay

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}\mathrm{He}^{2}$$

$$(\text{Parent} \rightarrow \text{Daughter} + \alpha)$$

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha$$

$$T_D + T_\alpha = \underbrace{\left(M_P - M_D - M_\alpha\right)}c^2 = \Delta M c^2 \equiv Q$$

Atomic masses can be used since the electron masses will cancel.

Kinematics

Treating the decay products nonrelativistically:

Momentum: $M_D v_D = M_\alpha v_\alpha \Rightarrow v_D = \frac{M_\alpha}{M_D} v_\alpha$

$$\text{Energy: } T_D + T_\alpha = Q = \frac{1}{2} M_D v_D^2 + \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_\alpha v_\alpha^2 \left(\frac{M_\alpha}{M_D} + 1 \right)$$

$$\Rightarrow T_\alpha = \frac{M_D}{M_\alpha + M_D} Q \quad \text{and} \quad T_D = \frac{M_\alpha}{M_\alpha + M_D} Q$$

- For a heavy nucleus, most of the kinetic energy released goes to the alpha.
- For this two-body decay, the alpha energy is unique (i.e. completely determined by the masses of parent and daughter).

Nuclei have Discrete Energy Levels

- Precise measurements have revealed that the emitted α's have a spectrum of discrete energies.
- This can be explained by assuming that the daughter nucleus can be left in an excited state, which subsequently decays:

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{*Z-2} + {}^{4}\mathrm{He}^{2}$$

$$^{A-4}Y^{*Z-2} \rightarrow {}^{A-4}Y^{Z-2} + \gamma$$

- The energy of the excited state can be determined from the α energy …

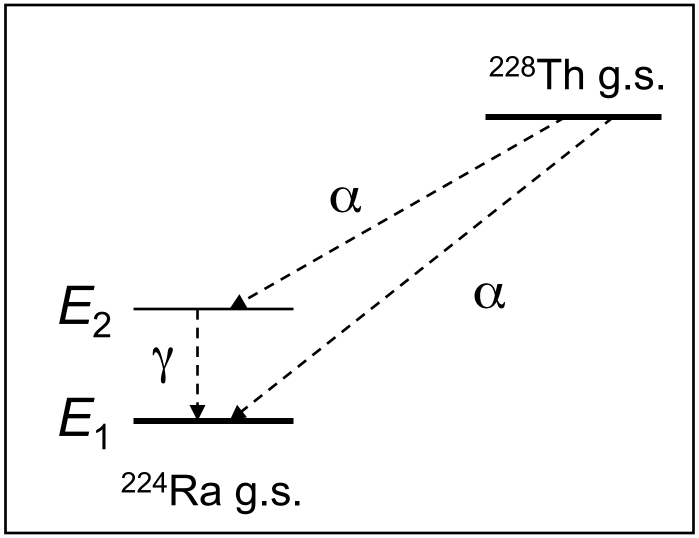
Example

- For a heavy nucleus, we can write:

$$T_\alpha = \frac{M_D}{M_\alpha + M_D} Q \approx \frac{A-4}{A} Q$$

- So, for ^{228}Th → ^{224}Ra + α:

$$Q \approx \frac{A}{A-4} T_\alpha = \frac{228}{224} T_\alpha$$

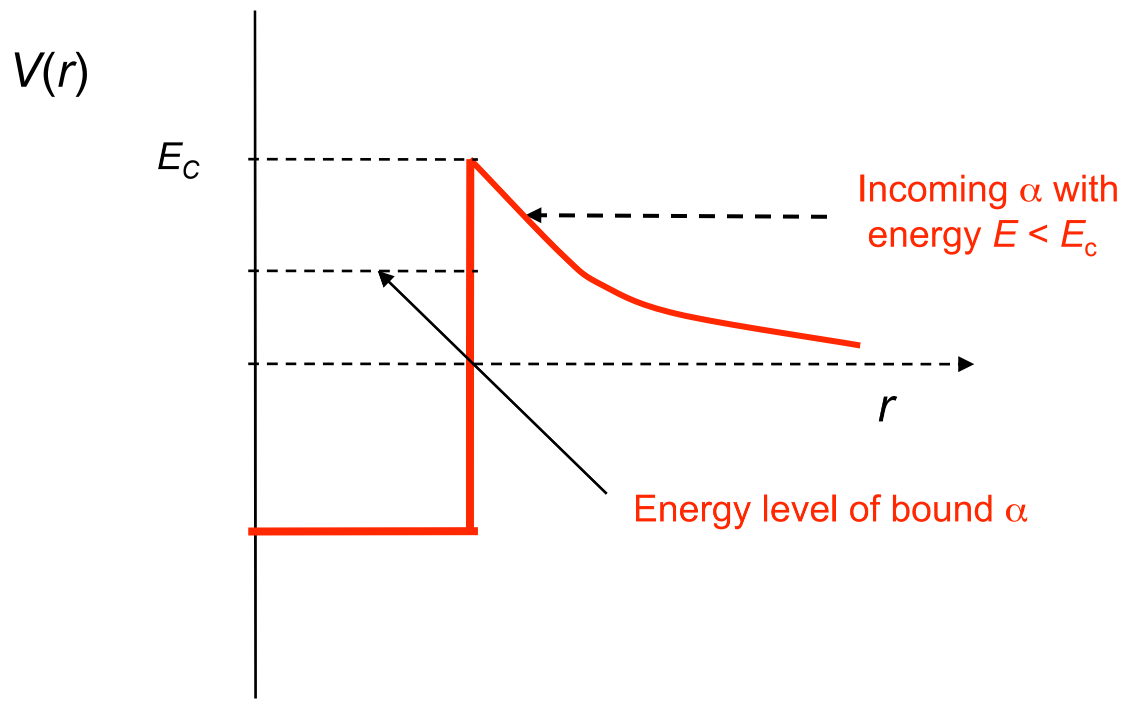


- If the highest energy α's have: T_α = 5.421 MeV and 5.338 MeV, the highest energy corresponds to the ground state of ^{224}Ra and the first excited state has energy:

$$E = E_2 - E_1 = Q_1 - Q_2 \approx \frac{228}{224}(5.421 - 5.338)\ \text{MeV} = 0.084\ \text{MeV}$$

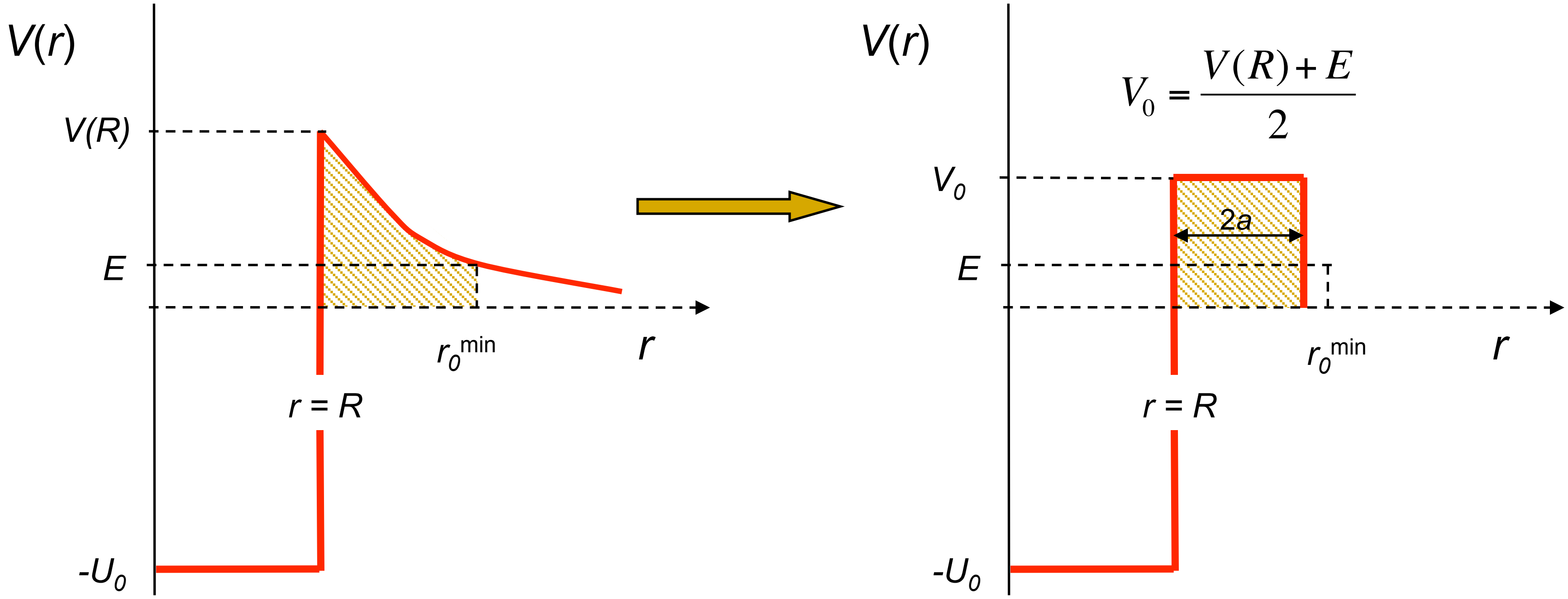
Barrier Penetration

- Low energy α's incident on heavy nuclei cannot surmount Coulomb barrier and will not be absorbed.
- However, comparable energy α's are emitted from such nuclei, during α-decay. How can this be?
- Answer: QM tunneling + a very large number of "attempts".



Simplified Potential

- Ignore angular dependence in S.E. and treat as 1-D problem.
- Replace Coulomb potential by square barrier of equal area.



Transmission Probability

$$T = \frac{\dfrac{4k_1 k}{(k_1+k)^2}}{1+\left[1+\left(\dfrac{\kappa^2 - k_1 k}{\kappa(k_1+k)}\right)^2\right]\sinh^2 2\kappa a} \approx \frac{4(V_0-E)}{V_0}\left(\frac{E}{E+U_0}\right)^{1/2}\left[4e^{-\frac{4a}{\hbar}\left[2M_\alpha(V_0-E)\right]^{1/2}}\right]$$

$$k_1^2 >> \kappa^2$$
$$k_1^2 >> k^2$$
$$2\kappa a >> 1$$

where $k_1 = \left[\dfrac{2M_\alpha}{\hbar^2}(E+U_0)\right]^{1/2}$

$$k = \left[\frac{2M_\alpha}{\hbar^2}E\right]^{1/2}$$

$$\kappa = \left[\frac{2M_\alpha}{\hbar^2}(V_0-E)\right]^{1/2}$$

Numerical Results

- For E = 4 MeV, V_0 = 14 MeV, U_0 = 40 MeV, $2a$ = 33 fm: $T \approx 7\times10^{-40} \Rightarrow$ there is little chance for α absorption by heavy nuclei.
- For E = 4 MeV, $T_\alpha = U_0 + E$ = 44 MeV $\Rightarrow v_\alpha \approx 0.15\ c$
- For $R \approx 10^{-12}$ cm, $v_\alpha/R \approx 4.5\times10^{21}$ / sec
- The rate of α emission is: $T\times\ v_\alpha/R \approx 3.2\times10^{-18}$ / sec
- The mean lifetime is the reciprocal of this decay rate: $\tau \approx 3.2\times10^{17}$ sec = 1.0×10^{10} yr
- Though the calculation was crude, the actual value is quite close to this estimate.

Beta Decay

- Nuclei with *N*/*Z* off the stability line, can undergo β-decay, converting a neutron to a proton or vice versa:

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}$$

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu$$

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu$$

- The fundamental decay processes are, respectively:

$$n \rightarrow p + e^{-} + \bar{\nu} \quad \text{(can also occur for a free neutron)}$$

$$p \rightarrow n + e^{+} + \nu \quad \text{(can't occur for a free proton)}$$

$$p + e^{-} \rightarrow n + \nu \quad \text{(electron capture)}$$

Need for the Neutrino

- Only the electron and daughter nucleus were actually observed in the decay:

$$^{A}X^{Z} \rightarrow {^{A}Y^{Z+1}} + e^{-}$$

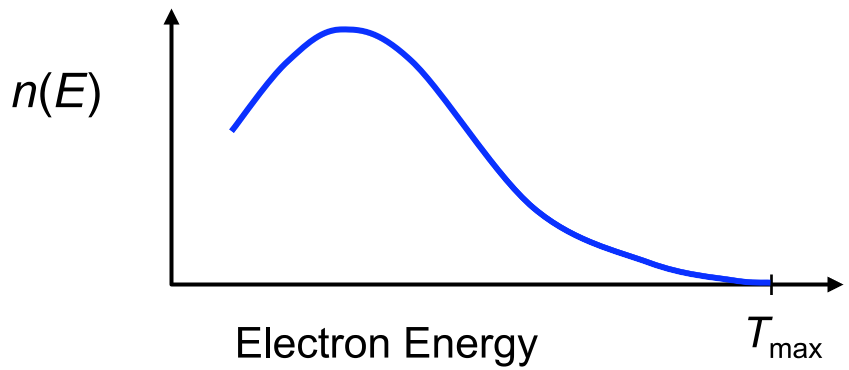
- Energy conservation requires:

$$E_X = M_X c^2 = E_Y + E_e = T_Y + M_Y c^2 + T_e + m_e c^2$$

$$\Rightarrow T_e = \left(M_X - M_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Such a two-body decay will give a fixed energy for the electron, but …

Need for the Neutrino, cont'd.



- The electron energy spectrum is *continuous* with a maximum value corresponding to the above two-body decay analysis. Energy conservation is at stake!
- Also, angular momentum conservation cannot be satisfied in the two-body decay: the number of nucleons does not change, but a spin-1/2 electron is emitted as a result of the decay.
- Pauli proposed an unseen "**neutrino**", which carries off energy, has spin =1/2, and which does not interact with matter appreciably [an essentially massless (since the endpoint energy corresponds with two-body decay) neutral particle].

The Antineutrino

- The neutrino has an antiparticle. Unlike other particles, the neutrino appears to be pointlike, uncharged and has no magnetic moment or nucleon number. So what distinguishes it from its antiparticle?
- Helicity (handedness): for a massless particle, the component of the spin along the direction of motion.
  - Electrons are accompanied by right-handed (positive helicity) antineutrinos.
  - Positrons are accompanied by left-handed (negative helicity) neutrinos.
- Recent experiments indicate the neutrino, in fact, has a small mass. This has important implications, as we'll see later.

Kinematics for Beta-Decay

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \overline{\nu}$$

(Parent $\rightarrow$ Daughter + e + $\overline{\nu}$)

- Energy conservation gives:

$$T_D + T_e + T_\nu = \left(M_P - M_D - m_e - m_\nu\right)c^2 = \Delta M c^2 = Q$$

- The decay can occur provided:

$$Q \approx \big[\underbrace{M(A,Z) - M(A,Z+1)}_{\text{Atomic masses}}\big]c^2 \geq 0$$

- For a heavy nucleus, we can neglect T_D:

$$T_e + T_\nu \approx Q \Rightarrow 0 \leq T_e \leq Q$$

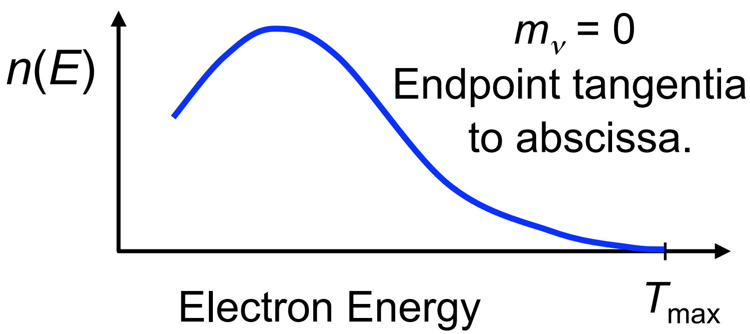
Lepton Number

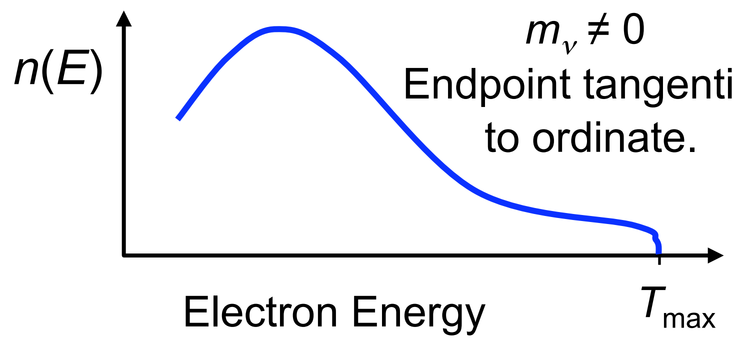
- Electrons and neutrinos are **leptons**, belonging to a family of leptons:

$$(e^-, \nu_e),\ (\mu^-, \nu_\mu),\ (\tau^-, \nu_\tau)$$

- All leptons have lepton number +1
- All their antiparticles have lepton number -1.
- Lepton number, like baryon number, appears to be conserved in all processes.
- Further, each type of lepton is produced with the corresponding type of neutrino.

Neutrino Mass





- The shape of the β-decay spectrum near the endpoint is sensitive to the neutrino mass.
- This requires very good experimental energy resolution.
- Current data are somewhat inconsistent, but Katrin (Karlsruhe Tritium Neutrino Experiment) promises to be a major improvement.

Solar Neutrino Problem and Resolution

- Solar neutrinos are of type ν_e.
- Number of ν_e neutrinos detected on Earth was too small by a factor of 2-3 compared with solar models (Davis and Koshiba, - Nobel Prize in Physics, 2002).
- 1998: Super-Kamiokande (Japan).
  - Neutrinos of type ν_μ produced by cosmic rays hitting atmosphere.
  - Number of ν_μ detected on Earth depends on distance of production (i.e. overhead or beneath horizon). Evidence for *neutrino oscillations*, i.e. neutrinos changing flavors.
- 2002: Sudbury Neutrino Observatory (SNO).
  - Using heavy water (deuterium nuclei), the detector is sensitive to all neutrino flavors.
  - The total number of neutrinos detected agreed with the solar models. Further evidence for oscillations.

Neutrino Oscillations

- Neutrino mixing: For non-zero neutrino masses, the flavor eigenstates (i.e. *e*, μ, τ) and mass eigenstates are different:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

Flavor eigenstate

Mass eigenstate

- The mass eigenstates propagate with a phase related to the energy, and therefore mass, of the neutrino:

$$e^{-i\phi} = e^{-iEt/\hbar}$$

- The different masses propagate with different frequencies and so the mass content changes.
- This implies that the flavor also changes or *oscillates*.

Neutrino Mass and Dark Matter

- Mechanism for neutrino mass generation is currently a controversial topic. It requires some modification to the **Standard Model**.
  - Heavy right-handed neutrinos can induce mass in the light, ordinary (left-handed) neutrinos (*see-saw* mechanism). The mass of the light neutrino is inversely proportional to the mass of the heavy neutrino.
  - Certain *supersymmetric* theories can account for finite neutrino mass, but typically predict proton decay inconsistent with experiment.
- Neutrinos are extremely abundant in the universe, and a finite mass would contribute to the **dark matter**, needed to explain various cosmological anomalies.

The Weak Interaction

- Neutrons decay with a lifetime of ~900 sec:

$$n \rightarrow p + e^- + \bar{\nu}$$

  - Time scales for nuclear processes: ~ 10^{-23} sec
  - Time scales for EM processes: ~ 10^{-16} sec
- Fermi postulated a new force: “weak” force.
  - Must be weak to explain long lifetime of neutron.
  - Must be short-ranged since it occurs within nuclei.
- Relative strengths of forces:

  $1 : 10^{-2} : 10^{-5} : 10^{-39}$

  (Strong, EM, Weak, Gravitational)

Fermi's Four-Fermion Theory

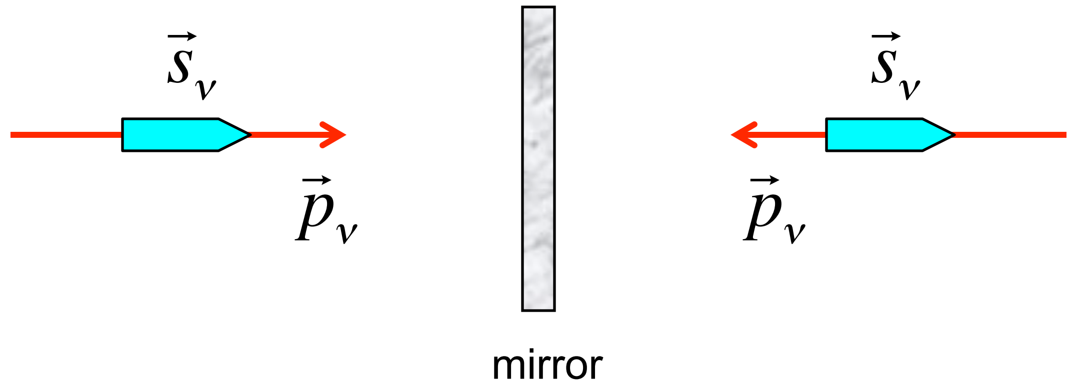
- Weak transitions are characterized by the weak Hamiltonian. The transition probability can be calculated using Fermi's Golden Rule:

$$P = \frac{2\pi}{\hbar} \left| H_{fi} \right|^2 \rho(E_f)$$

$$H_{fi} = \langle f | H_{wk} | i \rangle = \int d^3x \psi_f^*(x) H_{wk} \psi_i(x)$$

- The process $n \rightarrow p + e^- + \overline{\nu}$ connects four fermionic states.
- A large body of experiments put strict constraints on the nature of this four-fermion theory.

Parity Violation



- Under mirror (parity) inversion:

$$\vec{r} \to -\vec{r}$$

$$\vec{p} \to -\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} \to (-\vec{r}) \times (-\vec{p}) = \vec{L}$$

- The handedness therefore changes and left-handed neutrinos become right-handed.
- But right-handed neutrinos do not seem to exist, so the parity transformed process does not occur.
- Parity must be violated in weak interactions. Confirmed by C.S. Wu in 1956.

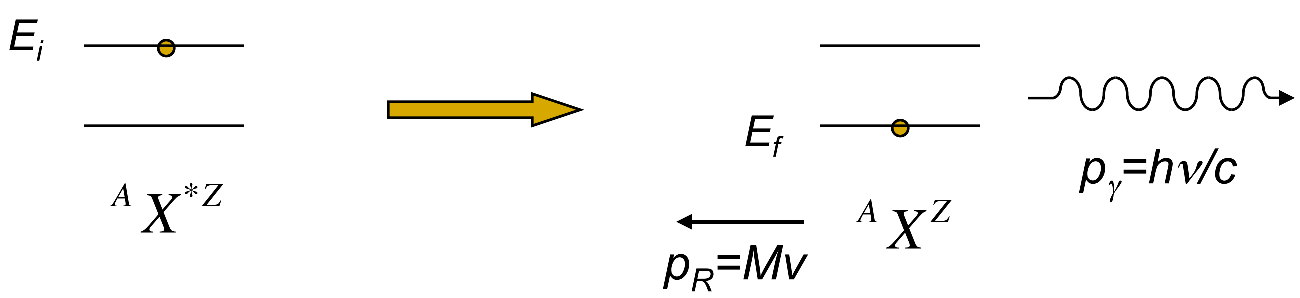
Gamma Decay

- Excited nuclei can de-excite through emission of a photon:

$$^{A}X^{*Z} \rightarrow {}^{A}X^{Z} + \gamma$$

- The process is electromagnetic.
  - The photon carries away at least one unit of angular momentum.
  - The decay conserves parity.
  - Lifetimes are typically ~10^{-16} sec.
  - Photon energies are typically ~100 keV.

Kinematics



Photon emission process (absorption can also occur for nucleus initially in its ground state).

For photon emission or absorption:

$$E_i = E_f \mp h\nu + \frac{1}{2}Mv^2 \quad \text{and} \quad \frac{h\nu}{c} = Mv$$

$$\Rightarrow h\nu = \mp\left(E_i - E_f - \frac{h^2\nu^2}{2Mc^2}\right) = \mp\left(E_i - E_f - \Delta E_R\right)$$

where $- \Rightarrow$ absorption and $+ \Rightarrow$ emission

Resonant absorption

- Can a photon emitted by one nucleus be absorbed by another of the same type?
  - If we can neglect recoil, then obviously YES.
  - Otherwise it would appear NO, since the emitted photon will have slightly less energy than the level spacing given some energy goes into recoil. Also, the absorbing nucleus must receive a slightly higher energy than the level spacing, since it too must recoil.
  - However, the level has a natural linewidth. So the question is: Is the linewidth larger or smaller than the recoil energy? …

Resonant Absorption, cont'd.

- Natural linewidth of an unstable level:

$$\delta E = \Gamma \approx \frac{\hbar}{\tau} \quad \text{where} \quad \tau = \text{ lifetime of state}$$

  - If $\Delta E_R \gg \Gamma$: resonant absorption cannot occur
  - If $\Delta E_R \ll \Gamma$: resonant absorption can occur
- Atoms and nuclei differ in this respect:
  - Atomic levels have longer lifetimes $\Rightarrow$ smaller Γ
  - Atomic transitions involve lower energy photons $\Rightarrow$ smaller ΔE_R
  - Which effect is larger?

Nuclear vs. Atomic Resonant Absorption

- Atoms (take A = 50, E_i - E_f = 1 eV, τ = 10^{-8} sec)

$$\Delta E_R = \frac{(h\nu)^2}{2Mc^2} \approx \frac{(1\ \text{eV})^2}{2 \times 50 \times 10^9\,\text{eV}} = 10^{-11}\ \text{eV}$$

$$\Gamma \approx \frac{\hbar}{\tau} \approx \frac{6.6 \times 10^{-16}\ \text{eV - sec}}{10^{-8}\ \text{sec}} = 6.6 \times 10^{-8}\ \text{eV}$$

Absorption occurs

- Nuclei (take A = 50, E_i - E_f = 10^5 eV, τ = 10^{-12} sec)

$$\Delta E_R = \frac{(h\nu)^2}{2Mc^2} \approx \frac{(10^5\ \text{eV})^2}{2 \times 50 \times 10^9\,\text{eV}} = 10^{-1}\ \text{eV}$$

$$\Gamma \approx \frac{\hbar}{\tau} \approx \frac{6.6 \times 10^{-16}\ \text{eV - sec}}{10^{-12}\ \text{sec}} = 6.6 \times 10^{-4}\ \text{eV}$$

Absorption does not occur

Mössbauer Effect

- Nuclear resonant absorption would occur if the recoil mass were much larger.
- Rudolf Mössbauer: embed the emitter and absorber in crystals.
  - The atom/nucleus is locked to the crystal $\Rightarrow$ the entire crystal recoils $\Rightarrow$ the recoil energy is negligible.
  - Energy levels have been measured to ~10^{-7} eV (1 part per 10^{12} !) .
  - Can use this technique to measure hyperfine splittings in nuclei.