CHAPTER 5
Applications of Nuclear Physics

Lecture Notes For
PHYS 415
Introduction to Nuclear and Particle Physics

To Accompany the Text
Introduction to Nuclear and Particle Physics, 2nd Ed.
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World Scientific

Nuclear Energy

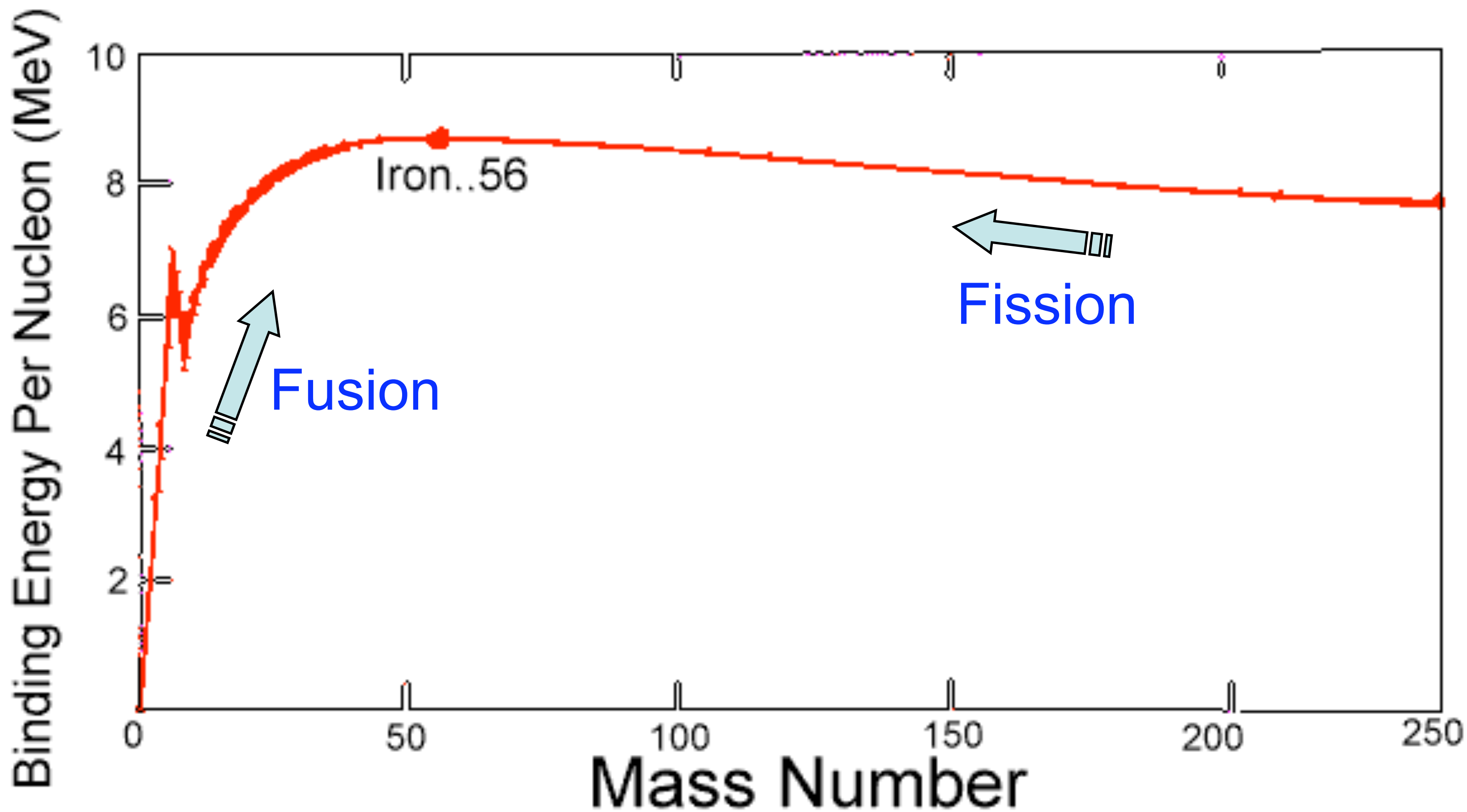
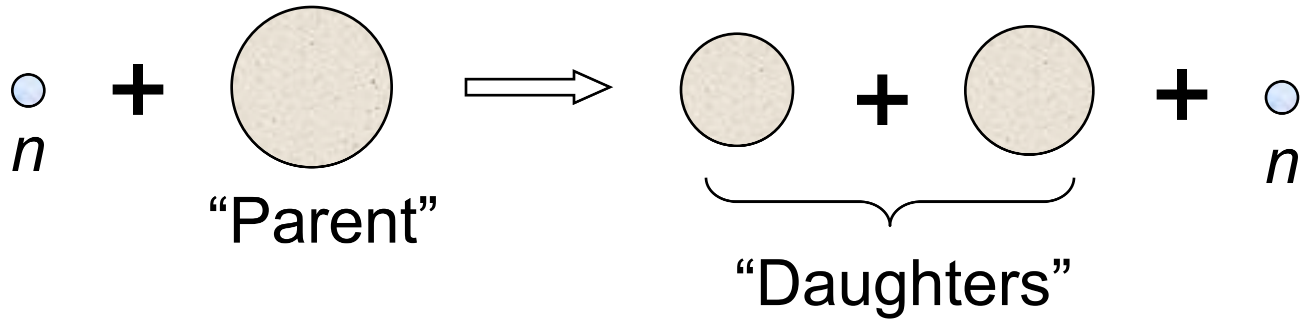


Figure from http://library.thinkquest.org/3471/mass_binding_body.html

Nuclear Fission



- Heavy nuclei can split (**fission**) into two smaller (daughter) nuclei when perturbed. These nuclei are more tightly bound than the parent nucleus.
- The mass difference between the parent and daughter nuclei shows up as kinetic energy of the reaction products.
- If one neutron is released on average, then this can give rise to a sustainable source of power: fission reactor.
- In some cases, there is more than one energetic neutron released. This can lead to a runaway chain reaction: fission bomb.

Energy Release

- Thermal neutrons ($T \approx 300$ K, $E \approx 1/40$ eV) can induce fission on some nuclei:

$$^{235}\mathrm{U} + n \rightarrow {}^{148}\mathrm{La} + {}^{87}\mathrm{Br} + n$$

- The energy release is given by the mass difference:

$$\left(m_{\mathrm{U}} - m_{\mathrm{La}} - m_{\mathrm{Br}}\right)c^2 = 178 \text{ MeV}$$

- Typical energy release: 0.9 MeV/nucleon
- Question: How much energy (in Joules) would be released by 1 g of ^{235}U in this process?

Liquid Drop Model

- An incident neutron can induce a deformation in the shape of the nucleus.
- Coulomb repulsion then drives the deformed shape to a two-lobe structure which eventually splits.
- The liquid drop model can be used to analyze the stability of the deformed nucleus.

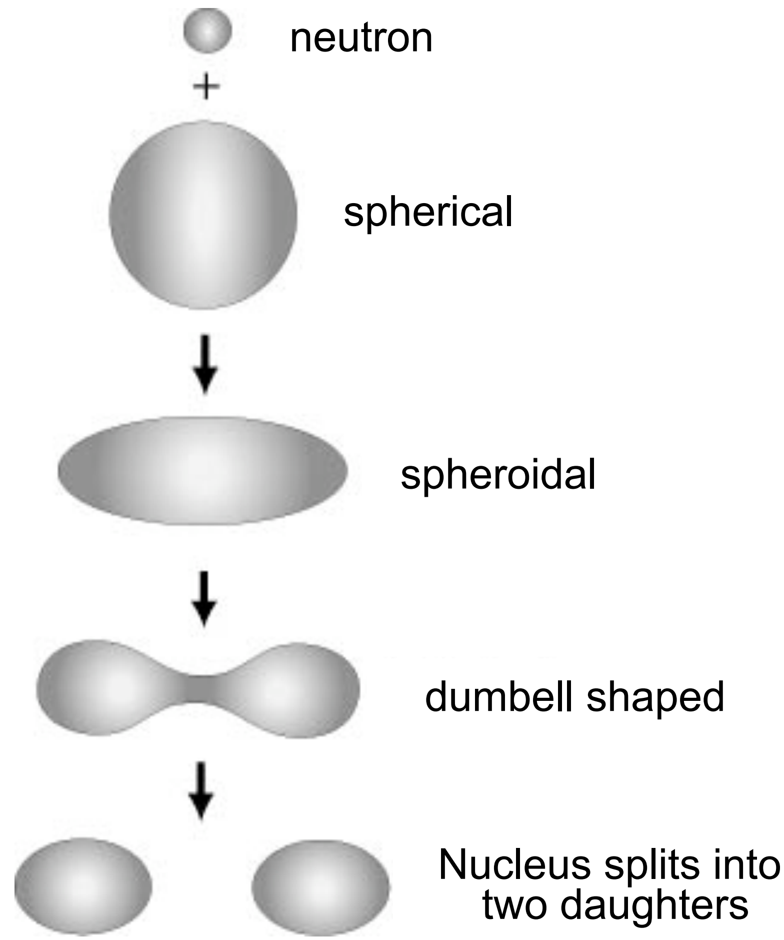
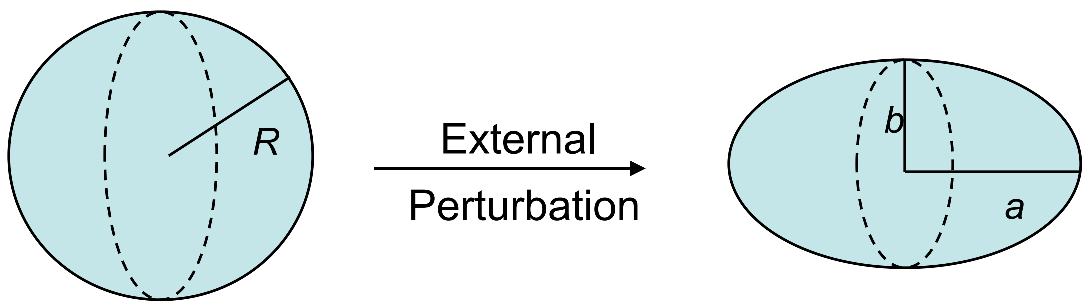


Figure adapted from:
http://www2.kutl.kyushu-u.ac.jp/seminar/MicroWorld3_E/3Part3_E/3P33_E/nuclear_fission_E.htm

Sphere vs. Ellipsoid



$$a = R(1+\varepsilon)$$

$$b = \frac{R}{\sqrt{1+\varepsilon}}$$

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi a b^2$$

volume is preserved

- We can use the semi-empirical mass formula to estimate the energies for each of the two shapes.
  - Consider the volume, surface and Coulomb energies.
  - Incompressible liquid $\Rightarrow$ volume energy is the same.

Comparison of Energies

- Surface energy: $$a_2 A^{2/3} \to a_2 A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)$$
- Coulomb energy: $$a_3 \frac{Z^2}{A^{1/3}} \to a_3 \frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon^2\right)$$
- The surface energy increases while the Coulomb energy decreases. The net change:

$$\Delta = \text{B.E. (ellipsoid)} - \text{B.E. (sphere)}$$

$$= \frac{1}{5}\varepsilon^2 A^{2/3}\left(2a_2 - a_3\frac{Z^2}{A}\right)$$

Criterion for Fission

$$\boxed{\begin{array}{l}\Delta > 0 \Rightarrow \text{sphere is stable against the perturbation} \\ \Delta < 0 \Rightarrow \text{sphere is unstable} \Rightarrow \text{fission}\end{array}}$$

- Fission will occur for $\dfrac{Z^2}{A} > \dfrac{2a_2}{a_3} \approx 47$
- But the "valley of stability" for large nuclei has $N \approx 1.7Z \Rightarrow Z^2/A \approx Z/2.7$
  - This criterion would only be satisfied for $Z > 127$ for nuclei in the valley of stability.
  - The spherical shape provides maximal binding and is stable.
- We must compare the binding energy of two fission fragments with that of the parent…

One Parent vs. Two Daughters

$$\Delta' = \mathrm{B.E.}\ (A,Z) - 2\ \mathrm{B.E.}\left(\frac{A}{2},\frac{Z}{2}\right)$$

$$= a_2 A^{2/3}\left(1-2\left(\frac{1}{2}\right)^{2/3}\right) + a_3\frac{Z^2}{A^{1/3}}\left(1-2\frac{\left(\frac{1}{2}\right)^{2}}{\left(\frac{1}{2}\right)^{1/3}}\right)$$

$$= a_2 A^{2/3}\left(1-2^{1/3}\right) + a_3\frac{Z^2}{A^{1/3}}\left(1-2^{-2/3}\right)$$

$$\approx 0.266 A^{2/3}\left(-16.4+\frac{Z^2}{A}\right)\ \mathrm{MeV}$$

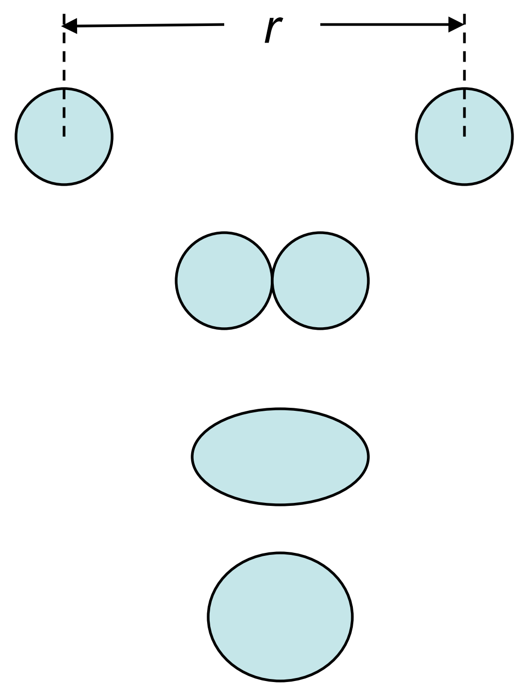
The daughters will be more tightly bound than the parent provided: $Z^2 > 16.4\,A$. In this case, even though the spherical parent may be stable, it's still energetically favorable to fission into two daughter nuclei.

Example

- For values of $A = 240$ and $Z = 92$:
  - $\Delta' \approx 200$ MeV (i.e. daughters more tightly bound)
  - If we assume that the daughters are identical, when they just touch their (repulsive) Coulomb energy will be ≈ 250 MeV (assuming they are both spherically symmetric distributions of charge, each of radius $r = 1.2 \text{ fm } A^{1/3}$).
  - The Coulomb barrier (or **activation energy**) would be ~ 50 MeV.
  - For asymmetrical daughters the Coulomb energy is significantly less. This is normally the case, resulting in activation energies of a few MeV.

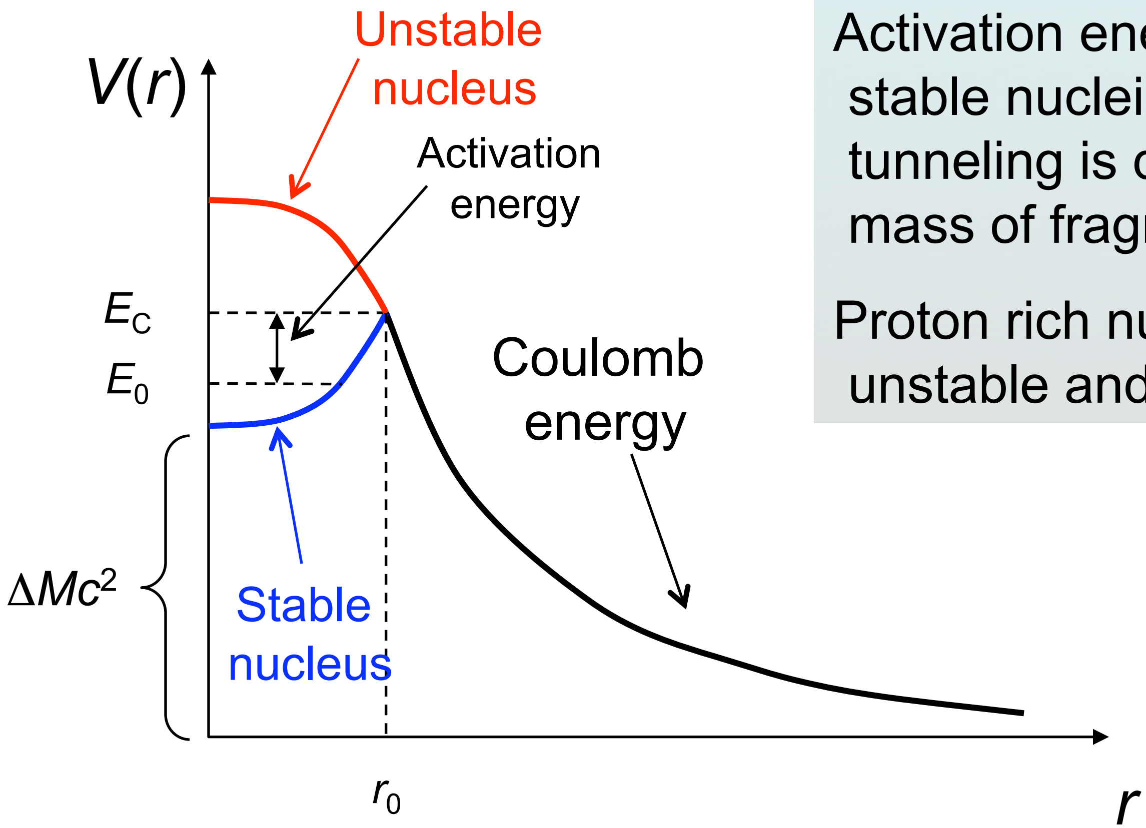
Energy vs. Separation Distance

Distance	Energy
large	0
r_0	$E_C \sim 250$ MeV
$\propto \varepsilon$	$E_{sphere} \pm k\varepsilon^2$
0	E_{sphere}



- The sign of the $k\varepsilon^2$ term depends on Z/A^2
- We can make a plot of the energy vs. distance …

Plot of Energy vs. Separation Distance



E_0 is due to quantum corrections.

Activation energy (few MeV) allows stable nuclei to decay. QM tunneling is difficult due to large mass of fragments.

Proton rich nuclei are inherently unstable and decay spontaneously.

^{235}U versus ^{238}U

- $^{235}\text{U} + n \rightarrow {}^{236}\text{U}$
  - Odd-even nucleus → even-even nucleus
  - Final nucleus is tightly bound: $\Delta M \approx 6.5$ MeV
  - Activation energy of ^{236}U ≈ 5 MeV
  - The mass difference provides more than enough energy to fission ⇒ even thermal neutrons can induce fission in ^{235}U.
- $^{238}\text{U} + n \rightarrow {}^{239}\text{U}$
  - Even-even nucleus → odd-even nucleus
  - Final nucleus is less tightly bound: $\Delta M \approx 4.8$ MeV
  - Activation energy of ^{239}U ≈ 6 MeV
  - Fission requires at least 1.2 MeV for the incident neutron.

Chain Reaction

- Typically, several neutrons are released along with the daughter nuclei. Each of these neutrons can induce additional fissions, leading to a **chain reaction**. Define:

$$k = \frac{\#\ \text{neutrons in } n+1 \text{ stage of fission}}{\#\ \text{neutrons in } n \text{ stage of fission}}$$

  - $k < 1$: *sub-critical*; no sustained reaction
  - $k = 1$: *critical*; sustained reaction ⇒ reactor
  - $k > 1$: *supercritical*; uncontrolled reaction ⇒ bomb

Fission Reactors

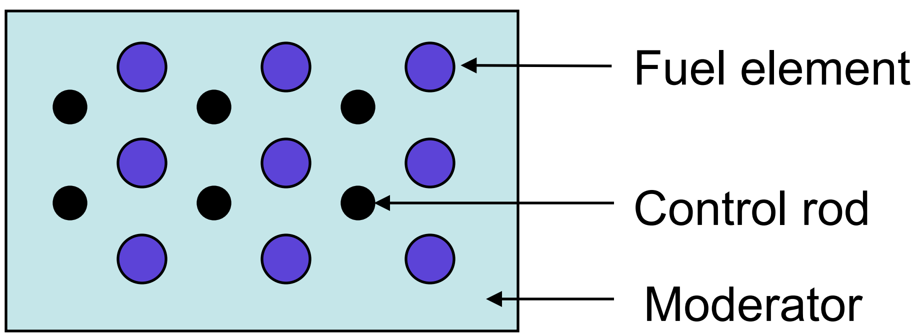
From http://en.wikipedia.org/wiki/Nuclear_reactor

Reactors use enriched Uranium:

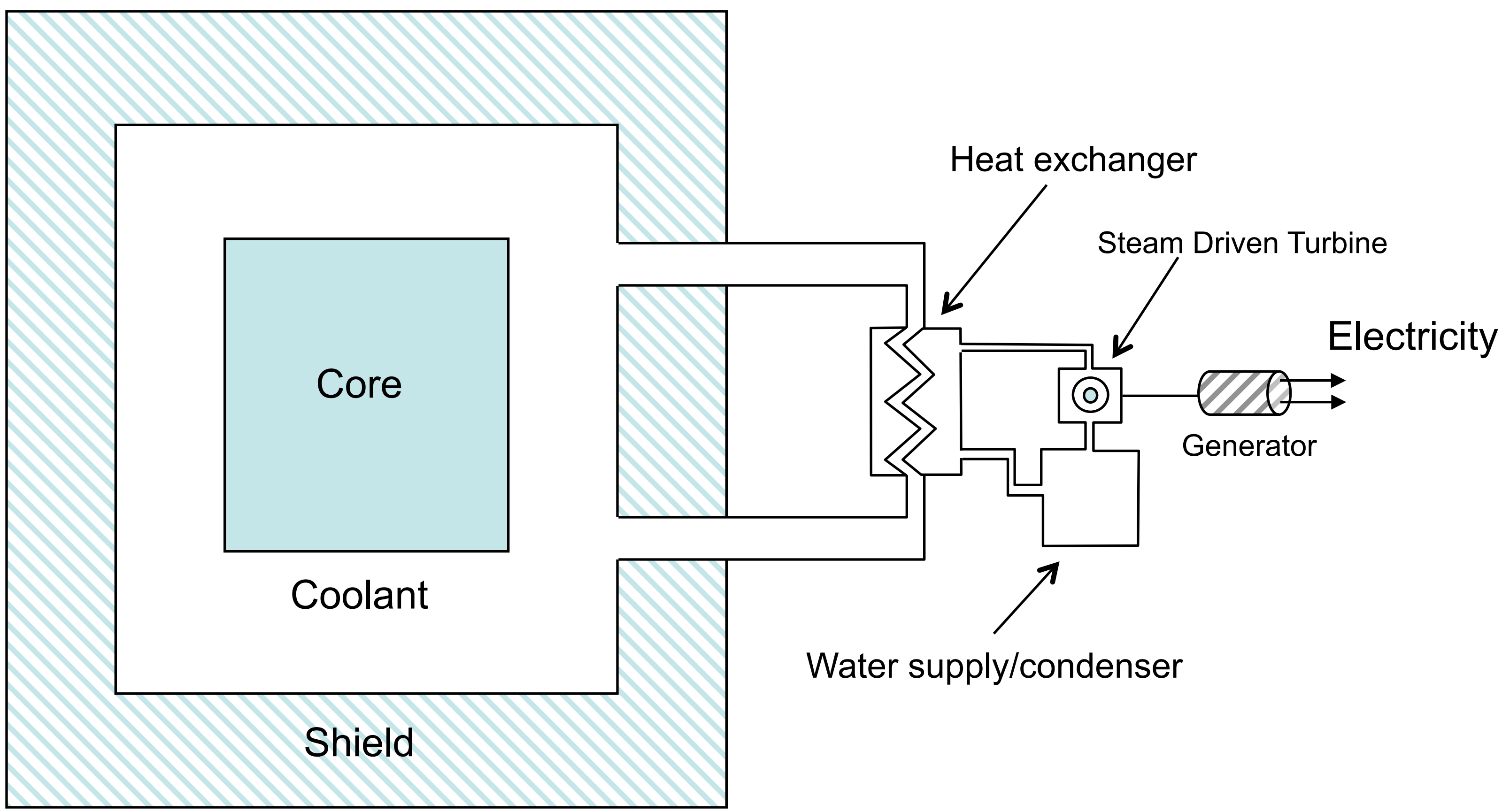
^{235}U lifetime ~ 7×10^8 yr; ^{238}U lifetime ~ 5×10^9 yr

Reactor Core

- Fuel elements are surrounded by a *moderator*.
  - Slows the fast neutrons, increasing probability for absorption.
  - Usually heavy water (D_2O) since deuterium has a low neutron absorption cross section compared to normal hydrogen.
- *Control rods*:
  - Usually made of Cadmium, which has a high neutron absorption cross section.
  - Can be inserted or removed to regulate the value of k.



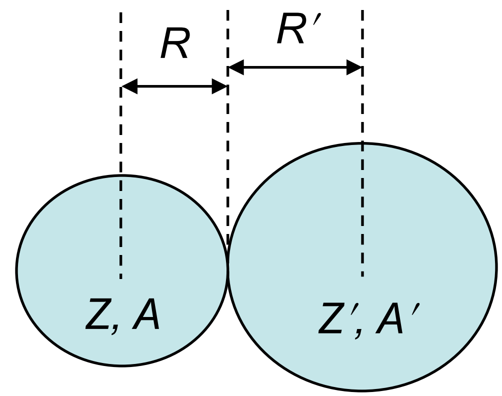
Nuclear Power Plant



Nuclear Fusion

- Light nuclei are less tightly bound than medium mass nuclei. Combining (fusing) two light nuclei will result in the release of energy.
  - Less nucleons per nucleus $\Rightarrow$ less energy released per fusion.
  - Needed light nuclei are readily abundant however.
  - This is the mechanism for energy production in the sun.
  - Must overcome the Coulomb barrier …

Coulomb Barrier



$$V_{\text{Coulomb}} = \frac{ZZ'e^2}{R+R'} = \frac{e^2}{\hbar c}\frac{\hbar c ZZ'}{1.2\ \text{fm}\left(A^{1/3}+A'^{1/3}\right)}$$

$$= \frac{1}{137}\frac{197\ \text{MeV - fm}}{1.2\ \text{fm}}\frac{ZZ'}{A^{1/3}+A'^{1/3}} \approx \frac{ZZ'}{A^{1/3}+A'^{1/3}}\ \text{MeV}$$

$$\xrightarrow{A\approx A'\approx 2Z\approx 2Z'} \frac{1}{8}A^{5/3}\ \text{MeV} \xrightarrow{A=8} 4\ \text{MeV}$$

Fusion Requires High Temperatures

- If the matter is hot enough, thermal motion can overcome the Coulomb barrier.
- Assuming each nucleus must have ~ 2 MeV kinetic energy, the temperature would be:

$$T = E/k = \frac{2\times 10^{6}\ \mathrm{eV}}{8.6\times 10^{-5}\ \mathrm{eV/K}} \approx 2\times 10^{10}\ \mathrm{K!}$$

- Even though this exceeds the temperature in the interior of stars, the tail of the Maxwell-Boltzmann distribution can provide the required energy.

Proton-Proton Cycle

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu_e + 0.42\ \text{MeV}$$

$$^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma + 5.49\ \text{MeV}$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2({}^{1}H) + 12.86\ \text{MeV}$$

The net effect is:

$$4({}^{1}H) \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + 2\gamma + 24.68\ \text{MeV}$$

e^{+}/e^{-} annihilation and photon energy conversion increase the total energy release per cycle.

CNO Cycle

$$3(^{4}He) \rightarrow {}^{12}C + 7.27\ \text{MeV}$$

And, subsequently:

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma + 1.95\ \text{MeV}$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e + 1.20\ \text{MeV}$$

$$^{13}N + {}^{1}H \rightarrow {}^{14}N + \gamma + 7.55\ \text{MeV}$$

$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma + 7.34\ \text{MeV}$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e + 1.68\ \text{MeV}$$

$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He + 4.96\ \text{MeV}$$

The net effect is:

$$4(^{1}H) \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + 3\gamma + 24.68\ \text{MeV}$$

Fusion Power

- Achieving a controlled, sustained fusion reaction is difficult. Three basic methods of confinement:
  - Gravitational: good for stars/galaxies, but not practical on Earth
  - Inertial: use laser/ion beams to compress fuel
  - Magnetic: confine and compress a hot, circulating plasma with magnetic fields
- One possible reaction is D + T:

  $^{2}H + {}^{3}H \rightarrow {}^{4}He + n + 17.6\ \mathrm{MeV}$
- Still have not maintained high temperatures for the needed product of ion density and confinement time (Lawson's criterion) necessary to achieve *break-even*.

Radioactive Decay

- Some nuclei are unstable and spontaneously undergo radioactive decay. The process is statistical, but the decay constant or mean life of a nucleus can be determined:

$$dN = N(t+dt) - N(t) = -N(t)\lambda dt$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

λ = decay constant
τ = mean life = $1/\lambda$
$t_{1/2}$ = half-life = $\tau \ln 2$

Activity

- **Activity** is defined as the number of radioactive decays per unit time:

$$A(t) = \left|\frac{dN}{dt}\right| = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

- One gram of ^{226}Ra ($t_{1/2}$ = 1620 yr) has an initial activity of 3.7×10^{10} /sec $\equiv$ 1 curie (Ci). This is a very large activity.
- An activity of 1/sec $\equiv$ 1 becquerel (Bq).

Poisson Statistics

- The number of decays in a time interval Δt:

$$\Delta N(t) = \int_{t-\frac{\Delta t}{2}}^{t+\frac{\Delta t}{2}} dt \mathrm{A}(t) = \tau \mathrm{A}(0) e^{-t/\tau}\left(e^{\frac{\Delta t}{2\tau}} - e^{-\frac{\Delta t}{2\tau}}\right)$$

$$\xrightarrow{\Delta t << \tau} \mathrm{A}(0)\Delta t e^{-t/\tau}$$

- Poisson statistics is the limiting case of the binomial distribution when the probability, p, of an occurrence (decay) is small but there is a large sample, N, contributing to the total number of occurrences. In our case: $p = \lambda\Delta t \ll 1$.
  - Mean number of decays in time $\Delta t = Np = \Delta N$
  - Standard deviation of mean number of decays = $(\Delta N)^{1/2}$
  - So the number of observed decays will be $\Delta N \pm (\Delta N)^{1/2}$

Radioactive Equilibrium

- The decay of a parent may produce an unstable daughter:
  Parent → Daughter_1 → Daughter_2 → Daughter_3 → …
- Assuming the parent is long-lived, relative to the daughters, we can solve for the steady-state condition. At this time, any given daughter nucleus will decay away at the same rate it is being fed from "above".

$$\Delta N_1 = -\lambda_1 N_1 \Delta t, \;\; \Delta N_2 = \lambda_1 N_1 \Delta t - \lambda_2 N_2 \Delta t, \;\; \Delta N_3 = \lambda_2 N_2 \Delta t - \lambda_3 N_3 \Delta t, \; \ldots$$

$$\Rightarrow \frac{dN_1}{dt} = -\lambda_1 N_1, \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2, \quad \frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3, \; \ldots$$

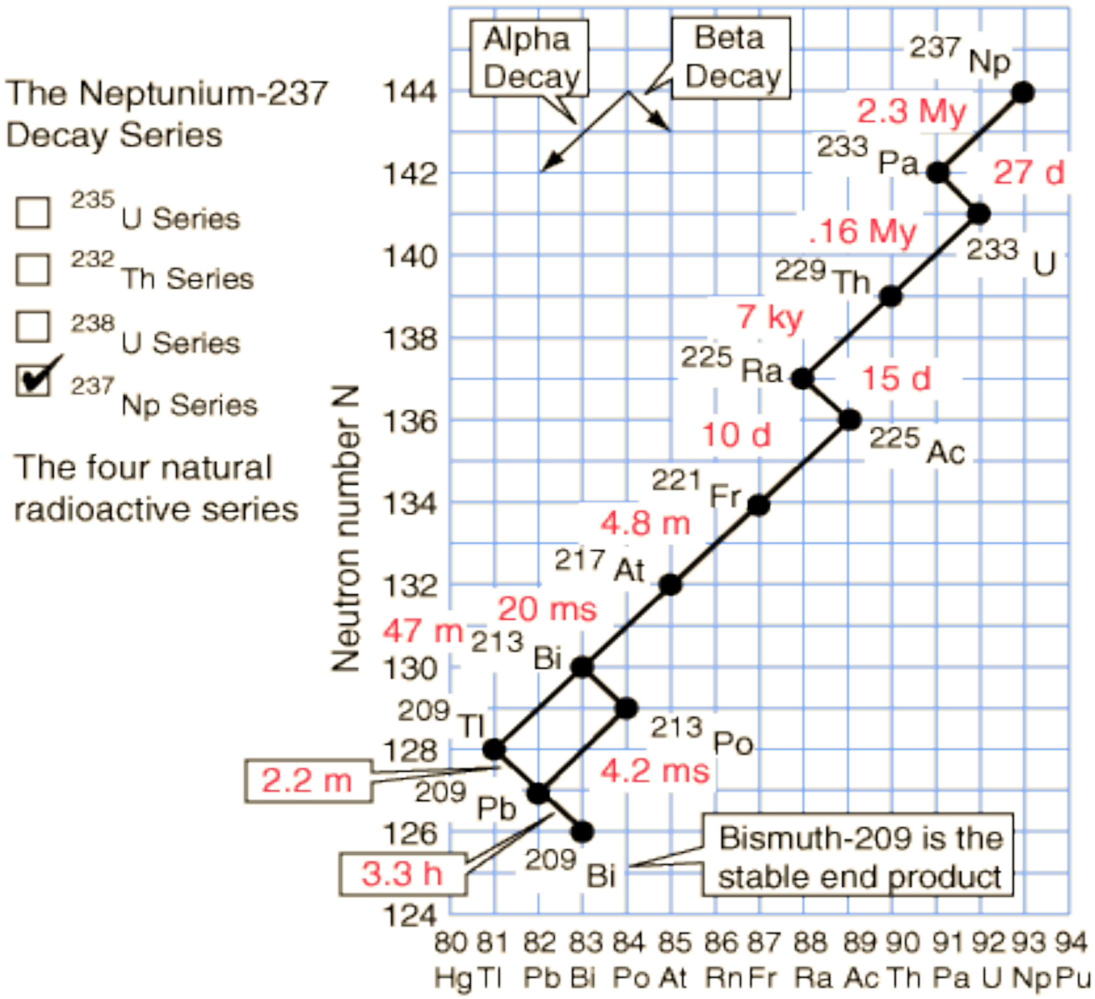
- *Secular equilibrium* is reached when all rates are zero. In this case, the number of each species will remain constant. This happens when:

$$\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3, \; \ldots$$

Natural Radioactivity

- There are roughly 60 naturally occurring radioactive nuclei, mostly with Z values between 81 and 92:
  - Substantial neutron excess.
  - Decay through a series of α and β emissions.
    - α emission increases the neutron excess and reduces A by four units.
    - β emission converts neutrons to protons, reducing neutron excess, but leaving A unchanged.
  - Decay chain terminates at a stable nucleus.
  - Four decay chains, labeled by long-lived parent (n = integer):
    - $A = 4n$ Thorium series
    - $A = 4n + 1$ Neptunium series
    - $A = 4n + 2$ Uranium-Radium series
    - $A = 4n + 3$ Uranium-Actinium series

Example: Neptunium Series



Due to the "short" half-life (2.3 My) ^{237}Np is not found on earth. The other three series have longer-lived parents which are observed on earth.

From: http://hyperphysics.phy-astr.gsu.edu/HBASE/nuclear/radser.html

Carbon Dating

- Cosmic rays incident on the earth's atmosphere can produce neutrons which are subsequently absorbed by nitrogen gas:

$$^{14}\mathrm{N} + n \rightarrow {}^{14}\mathrm{C} + p$$

- The nitrogen is thereby converted to Carbon-14 which is radioactive and subsequently decays:

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + e^- + \bar{\nu}_e \quad (t_{1/2} = 5730\ \mathrm{yr})$$

- Our atmosphere contains a small admixture of ^{14}C, in addition to the normal ^{12}C, both bound up in CO_2.
- Living organisms take up CO_2 and therefore have a mixture of both carbon isotopes.
- When the organism dies, CO_2 uptake stops. The ^{14}C decays away whereas the amount of ^{12}C remains fixed.
- We can deduce the age of a fossil by the activity of ^{14}C relative to a living organism.

Carbon Dating, cont'd.

- We can relate the activity (per gram) of a fossil *t* years after death of the organism to the activity (per gram) of a living organism (i.e. $t = 0$):

$$\mathrm{A}(t) = \left|\frac{dN}{dt}\right| = \lambda N(t) = \lambda N_0 e^{-\lambda t} = \mathrm{A}(0) e^{-\lambda t}$$

$$\lambda t = \ln \frac{\mathrm{A}(0)}{\mathrm{A}(t)} \Rightarrow t = \frac{1}{\lambda} \ln \frac{\mathrm{A}(0)}{\mathrm{A}(t)}$$

$$t = \frac{t_{1/2}}{\ln 2} \ln \frac{\mathrm{A}(0)}{\mathrm{A}(t)} \approx (8270 \text{ yr}) \times \ln \frac{\mathrm{A}(0)}{\mathrm{A}(t)}$$

- Currently, mass spectrometers can very accurately measure the differences in the concentration of the two carbon isotopes. However, corrections for nuclear testing and reactor accidents must be taken into account.