CHAPTER 6

Energy Deposition in Media

Lecture Notes For

PHYS 415

Introduction to Nuclear and Particle Physics

To Accompany the Text

Introduction to Nuclear and Particle Physics, 2nd Ed.

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World Scientific

Particle Detection

- In order to detect particles, they must leave some measurable trace: they must interact with matter.
- Charged particles interact mostly through the EM force.
  - Neutral particles are more difficult to detect.
  - Neutrinos, which interact only via the **weak** interaction are especially difficult to detect.

Stopping Power

- Charged particles transfer their energy to materials primarily through ionization and atomic excitations. The **stopping power** or ionization energy loss is:

$$S(T) = -\frac{dT}{dx} = n_{\text{ion}}\bar{I}$$

where n_{ion} = # electron - ion pairs per unit length

$\bar{I}$ = average energy needed to ionize an atom

- Nuclear collisions can also occur but with much smaller probability (nuclear cross sections are much smaller than atomic cross sections).

Bethe-Bloch Formula

- Hans Bethe and Felix Bloch derived an expression for energy loss of relativistic particles by ionization:

$$S(T) = \frac{4\pi Q^2 e^2 nZ}{m\beta^2 c^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{\bar{I}}\right) - \beta^2\right] \xrightarrow{\beta\text{ small}} \frac{4\pi Q^2 e^2 nZ}{m\beta^2 c^2}\ln\left(\frac{2mc^2\beta^2}{\bar{I}}\right)$$

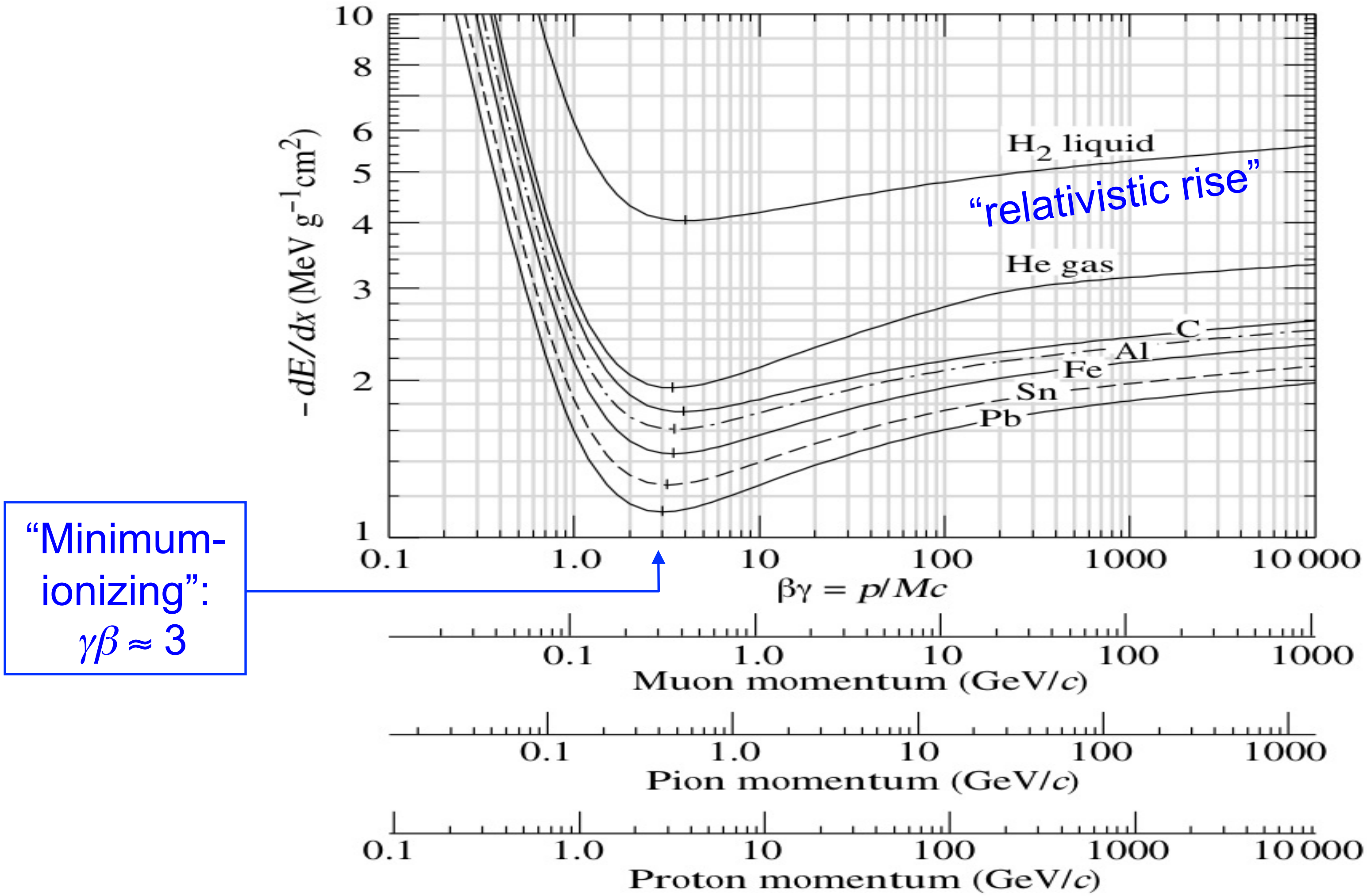
Incident particle:

$$\begin{cases} Q = ze = \text{ charge of incident particle} \\ \beta = \dfrac{v}{c} = \text{ velocity of incident particle compared to } c \\ \gamma = 1/\sqrt{1-\beta^2} \end{cases}$$

Material:

$$\begin{cases} m = \text{ electron mass} \\ Z = \text{ atomic number of medium} \\ n = \#\text{ atoms per unit volume } = \rho A_0 / A \\ \bar{I} = \text{ mean ionization/excitation energy of medium} \end{cases}$$

Energy Loss in Various Materials



From http://pdg.lbl.gov/2005/reviews/passagerpp.pdf

Minimum-Ionizing Particles

- For $\gamma\beta \approx 3$, particles are "minimum-ionizing".
- For very energetic particles, the "relativistic rise", caused by the γ^2 in the logarithmic term, saturates. Therefore, the stopping power of very energetic particles is not too different than the minimum ionizing value:

$$S \xrightarrow{\gamma\beta=3} S_{\min} \approx 0.33(13.7 - \ln Z)\rho\frac{Z}{A}\ \text{MeV/cm} \quad (\text{where we used } \bar{I} = 10Z\ \text{eV},\ z = 1)$$

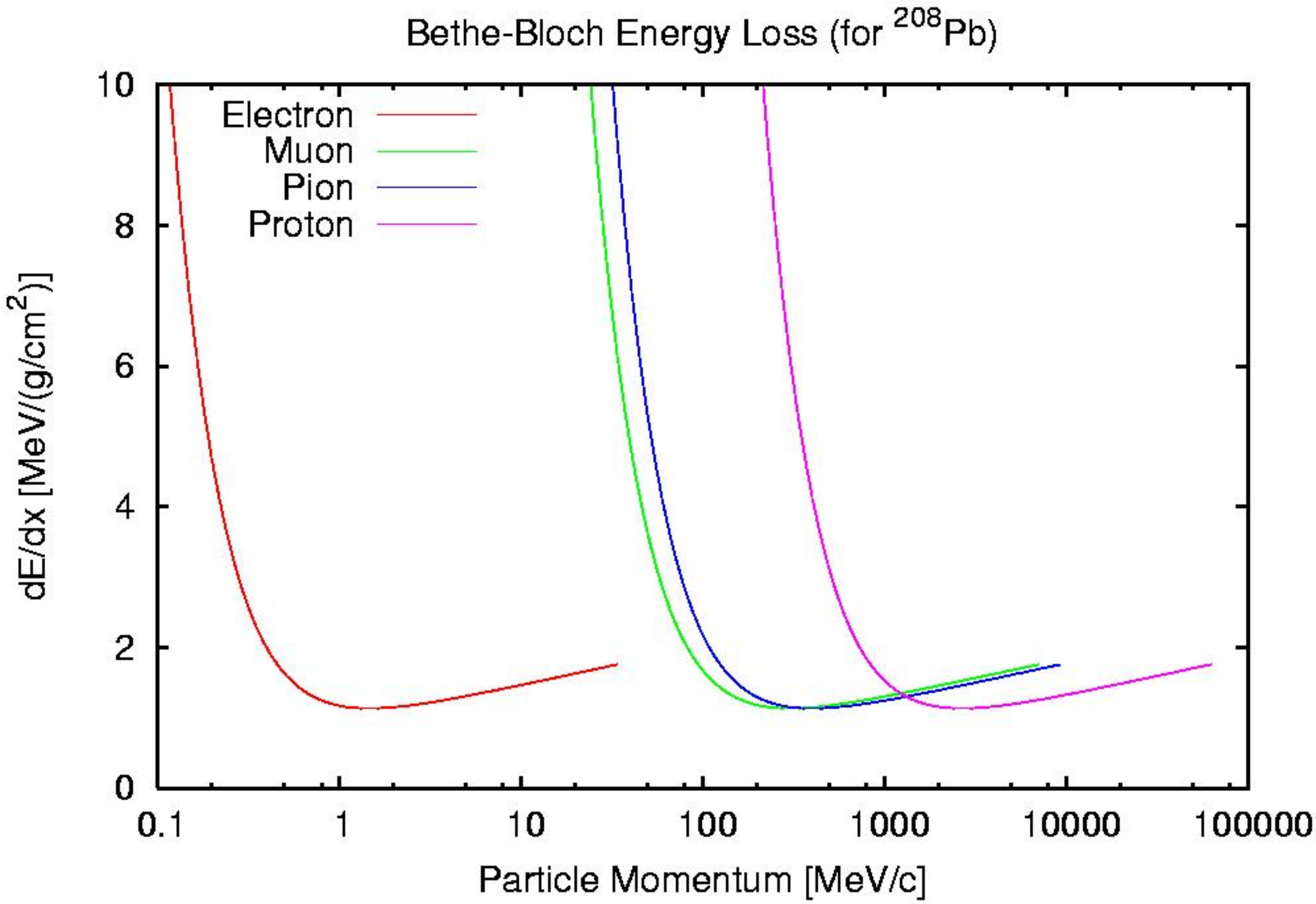
$$\text{for } Z = 20: \quad S_{\min} \approx 3.5\rho\frac{Z}{A}\ \text{MeV/cm} = 3.5\frac{Z}{A}\ \text{MeV/(gm/cm}^2) \approx \boxed{1.6\ \text{MeV/(gm/cm}^2)}$$

- Except for hydrogen, this is a good approximation.

Particle Identification

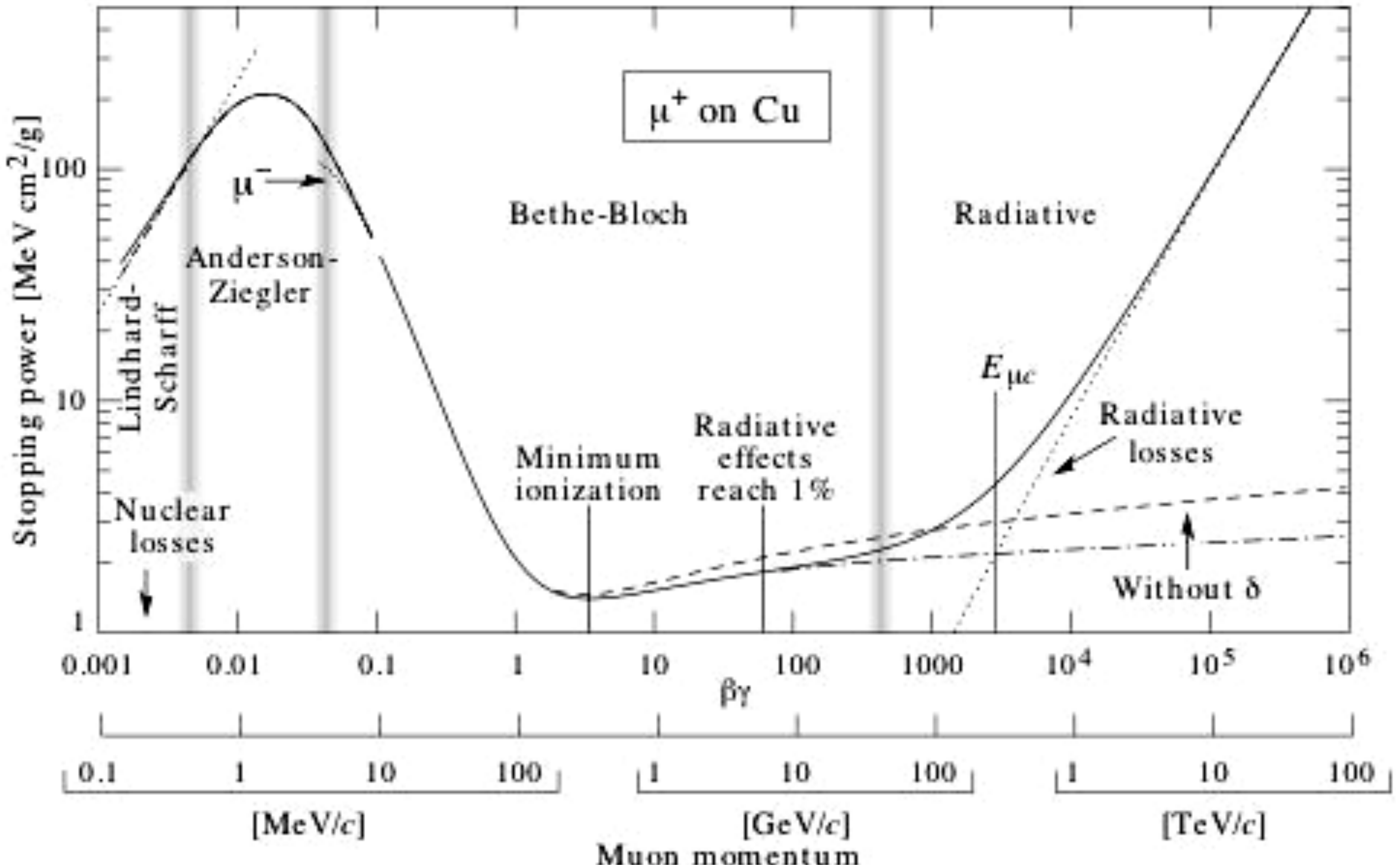
- Identifying the type of particle is one of the basic requirements of a detector system.
- Usually we can identify the particle from its mass.
- For low velocity particles, the energy loss depends strongly on mass.
- For fixed momentum, *p,* we can distinguish different mass particles by their different energy losses.
- This method does not work well at very high energies (i.e. when $v \to c$).

Bethe-Bloch and Particle ID



Note: Bethe-Bloch formula is not valid at very low and very high momenta. These curves have been truncated accordingly.

Energy Loss Mechanisms



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Particle Range

- If we know the stopping power vs. energy, we can determine the particle range:

$$R = \int_0^R dx = \int_0^T \frac{dx}{dT} dT = \int_0^T \frac{dT}{S(T)}$$

- For low energies, the range, just as the stopping power, depends strongly on particle mass.

Straggling

- Particles transfer their energy via collisions with atoms in the material. This is a statistical process and there is a range of possible energies for each collision.
- The energy loss after traversing a certain thickness of material will also reflect these statistical variations.
- Therefore, while the Bethe-Bloch and similar formulae give the mean energy loss, a given particle's energy loss will have a distribution about this mean value.
- This variation about the mean energy loss is called **straggling**.

Multiple Scattering

- Particles are also deflected through small angles via Coulomb (Rutherford) scattering with the atomic nuclei in a material. This, too, is a statistical process.
- The mean deflection is zero, since particles are equally likely to scatter to the left or right.
- A collection of particles, each undergoing many such interactions, will have a spread of angles, approximately Gaussian in form. After traversing a length *L* of material the *rms* width of the distribution is:

$$\theta_{\text{rms}} \approx \frac{20\ \text{MeV}}{\beta pc} z \sqrt{\frac{L}{X_0}} \quad \text{where } X_0 = \text{"radiation length" of material}$$

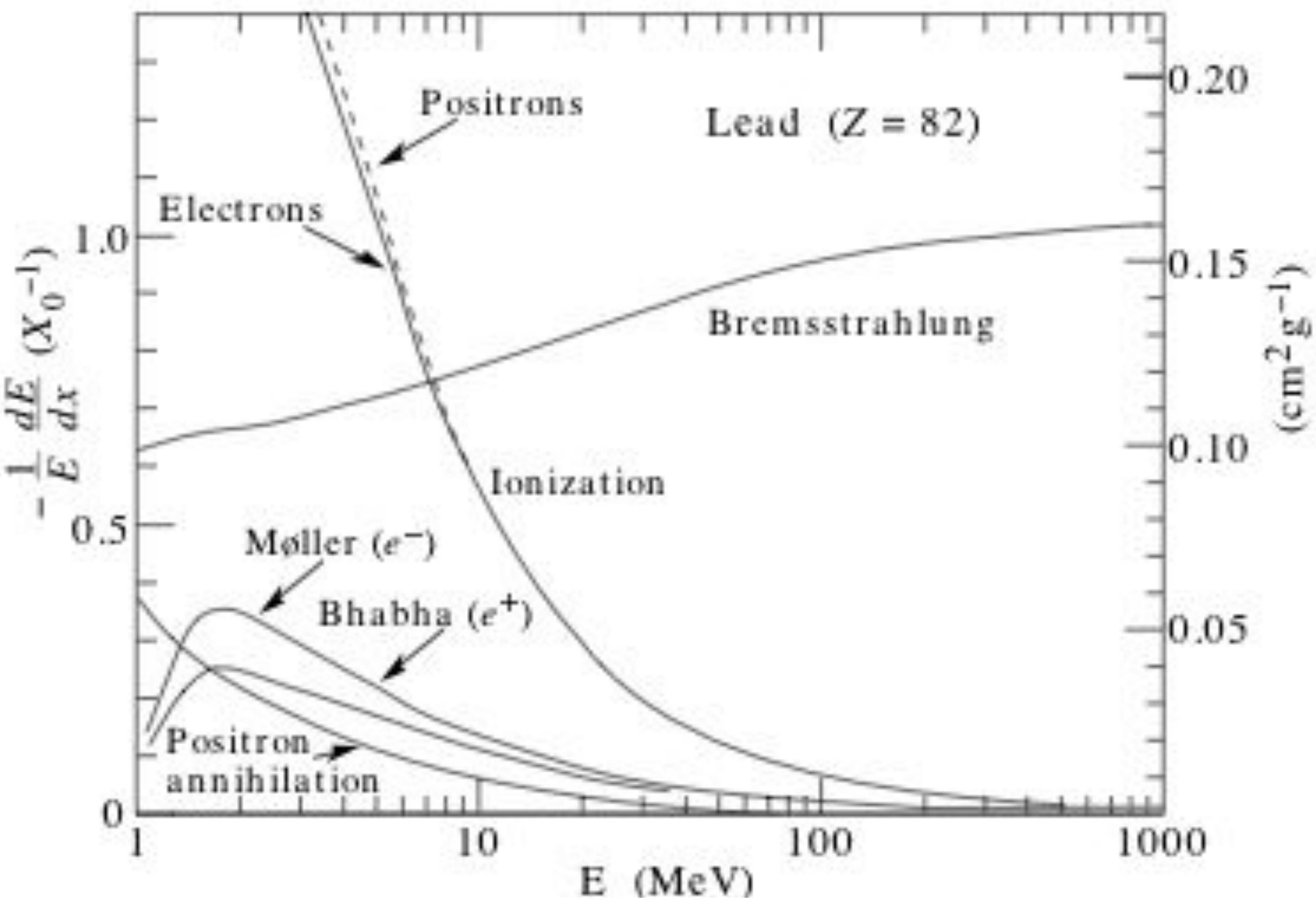
Energy Loss via Bremsstrahlung

- Electrons, due to their small mass, radiate photons readily in the electric fields of atomic electrons and nuclei.
- This becomes particularly important at high energies:
  - Ionization energy loss is nearly constant.
  - Bremsstrahlung rises roughly linearly with energy.

$$\left(-\frac{dT}{dx}\right)_{\text{tot}} = \left(-\frac{dT}{dx}\right)_{\text{ion}} + \left(-\frac{dT}{dx}\right)_{\text{brem}}$$

$$\text{and} \quad \frac{\left(\frac{dT}{dx}\right)_{\text{brem}}}{\left(\frac{dT}{dx}\right)_{\text{ion}}} \xrightarrow{T \text{ large}} \frac{TZ}{1200mc^2}$$

Bremsstrahlung vs. Ionization



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Radiation Length

- At high energies, the radiative energy loss is roughly proportional to energy:

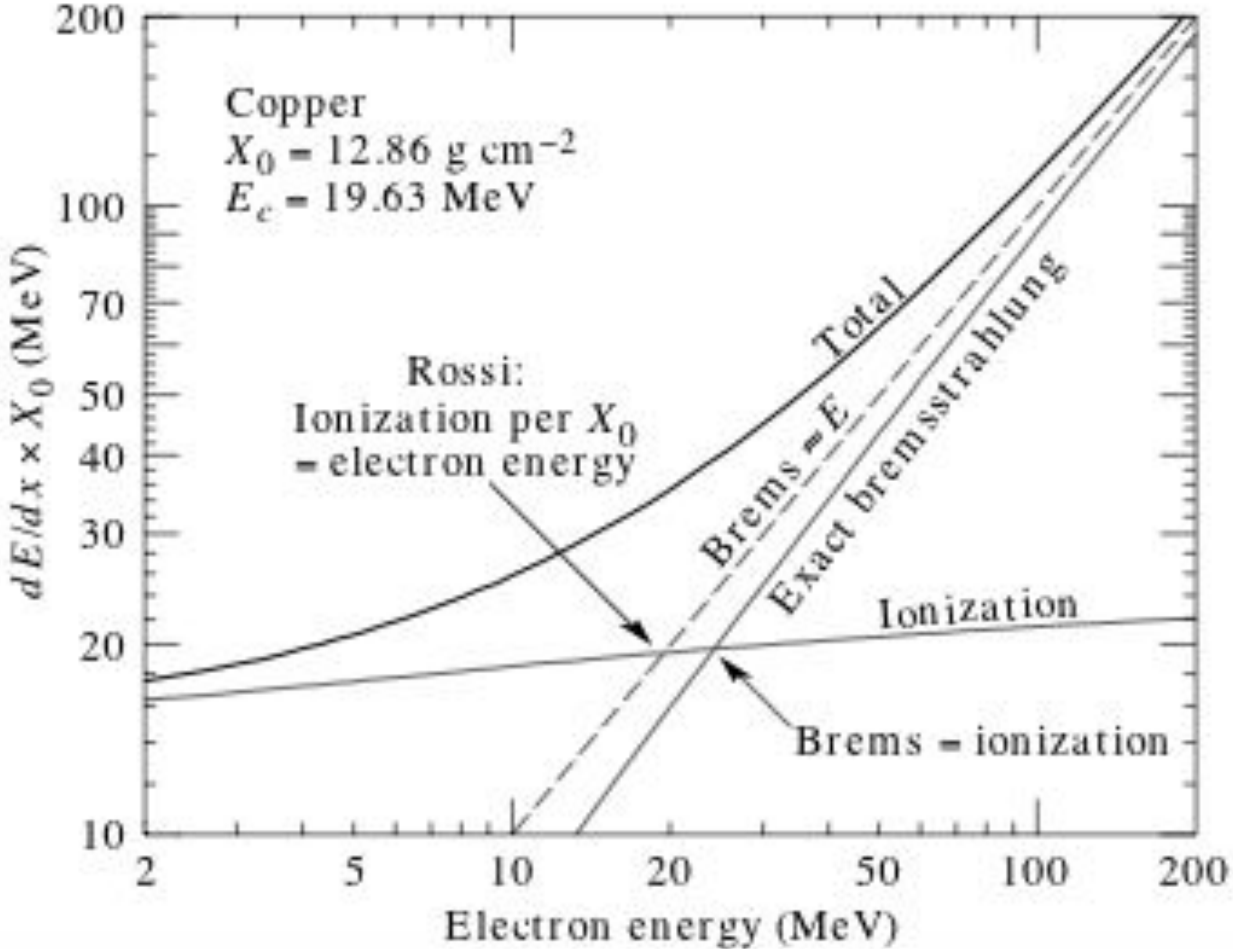
$$\left(\frac{dT}{dx}\right)_{\text{brem}} = -\frac{T}{X_0}, \quad \text{with } X_0 \approx 170\frac{A}{Z^2} \quad (\text{in gm/cm}^2)$$

$$\Rightarrow T = T_0 e^{-x/X_0}$$

- Define *critical energy*:

$$\text{At } T_c: \left(\frac{dT}{dx}\right)_{\text{brem}} = \left(\frac{dT}{dx}\right)_{\text{ion}} = -\frac{T_c}{X_0} \quad \text{and } T_c \approx \frac{600 \text{ MeV}}{Z}$$

Critical Energy



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Interactions of Photons with Matter

- Photons interact with matter in essentially three ways:
  - Photoelectric effect
    - Dominant at low energy
  - Compton scattering
    - Important at medium energy
  - Pair production
    - Dominant at high energy
- Can define the effective interaction (absorption) of photons via the *absorption coefficient*.

Absorption Coefficient, μ

- Define $I(x)$ = intensity of photons after traversing thickness x of material:

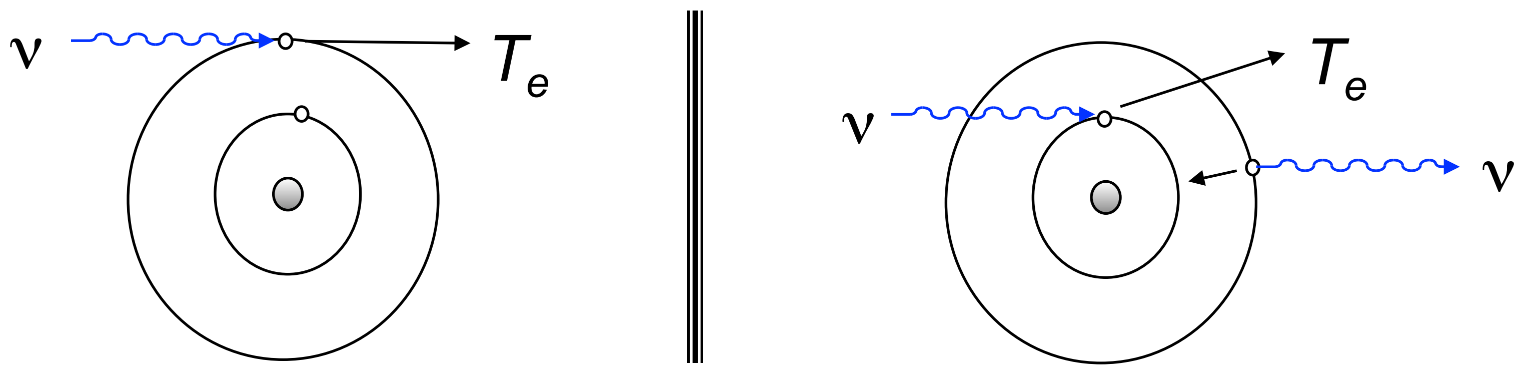
$$dI = I(x+dx) - I(x) = -\mu I(x)dx$$

$$\Rightarrow I(x) = I_0 e^{-\mu x}$$

$$\text{and "half-thickness"} = x_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$$

- The *mean free path* (= μ^{-1}) is the distance after which the intensity drops to 1/*e* of the initial value.

Photoelectric Effect

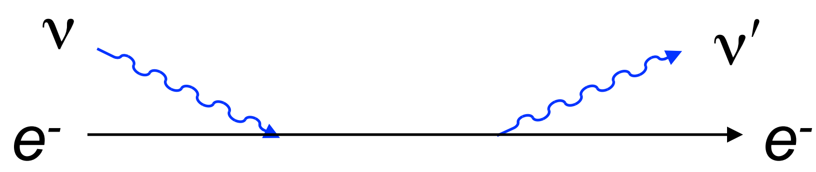


- For ionization energy I_B: $E_\gamma = h\nu = I_B + T_e$
- The cross section is particularly important for high Z atoms but is small above 1 MeV, scaling roughly as:

$$\sigma \propto \frac{Z^5}{(h\nu)^{7/2}} \quad \text{for } E_\gamma < m_e c^2$$

$$\sigma \propto \frac{Z^5}{h\nu} \quad \text{for } E_\gamma > m_e c^2$$

Compton Scattering



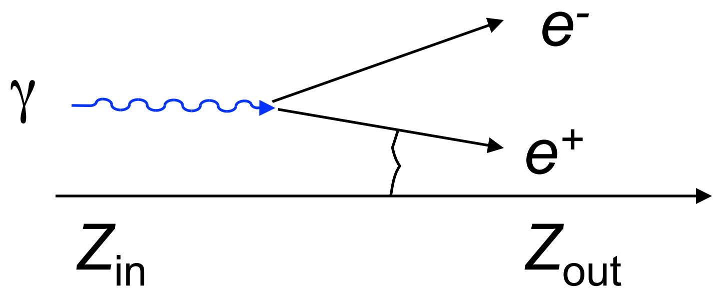
- Ignoring the electron binding energy (valid except for low energy photons) and treating the electron relativistically:

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{m_e c^2}\left(1 - \cos\theta\right)}$$

  where θ is the photon scattering angle.

- The cross section scales roughly as: $\sigma \propto \frac{Z}{h\nu}$

Pair Production



- Pair production requires a nucleus to recoil and conserve momentum. It cannot occur for an isolated photon.
- Neglecting small nuclear recoil energy, the threshold photon energy is:

$$E_\gamma \geq 2m_e c^2 \quad (\text{for } e^+e^- \text{ production})$$

- The cross section scales as Z^2 and above ~ 100 MeV, becomes essentially energy independent.

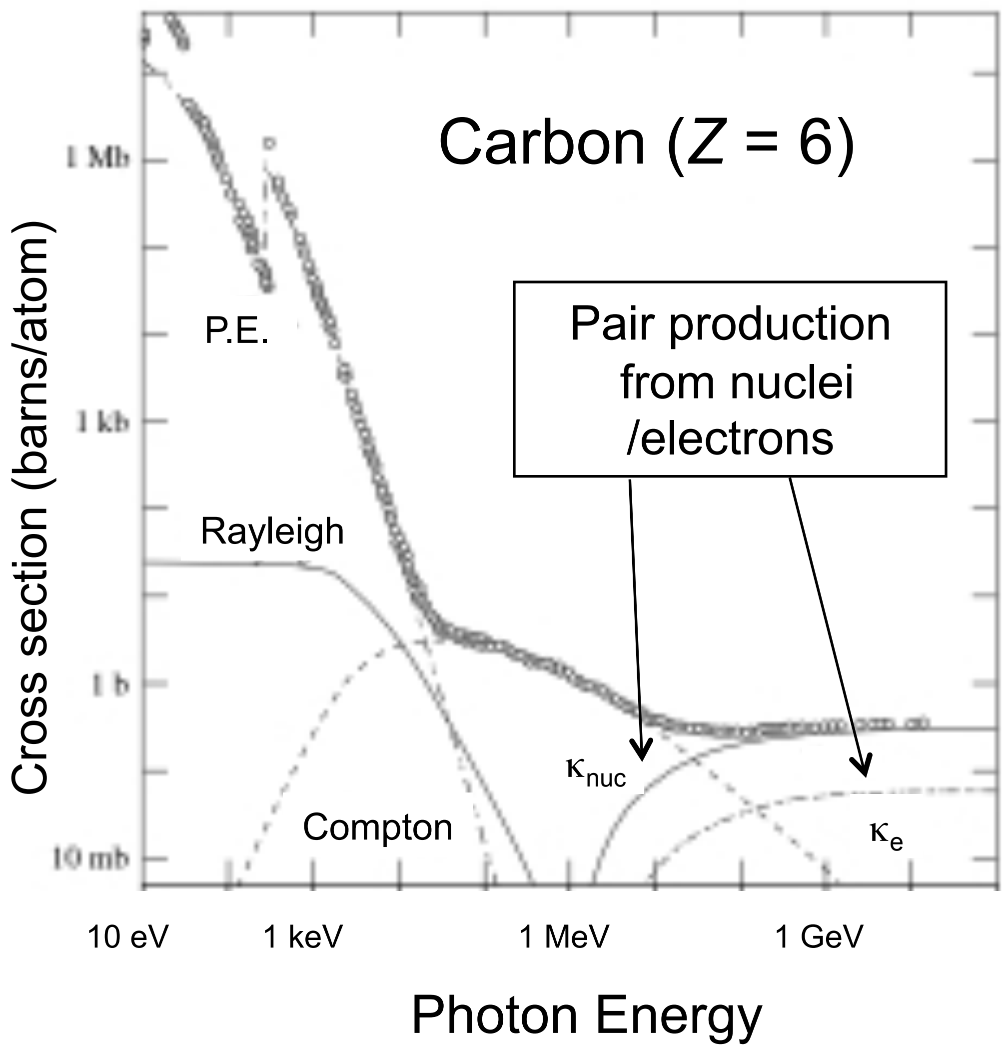
Pair Production at High Energies

- Since the cross section for pair production saturates at high energy, it can be characterized by a constant absorption length:

$$X_{\text{pair}} = \left(\mu_{\text{pair}}\right)^{-1} \approx \frac{9}{7} X_0$$

- At high energies pair production dominates, so the above formula characterizes the net effect of the medium on high energy photons.

Photon Interaction Cross Sections



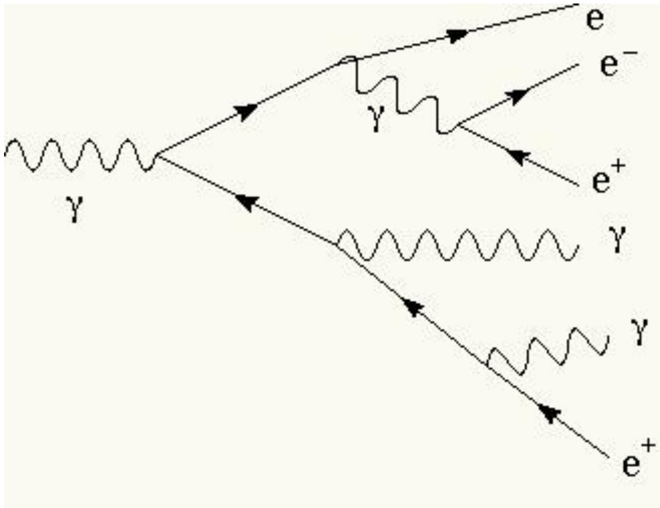
Adapted from http://pdg.lbl.gov/2005/reviews/passagerpp.pdf

Electron-Positron Annihilation

- The positrons, produced via pair production, interact with matter primarily through ionization and bremsstrahlung, just as electrons.
- The positrons, after losing most of their energy, form short-lived ($\sim 10^{-10}$ s) *positronium* "atoms", consisting of an electron and positron bound state.
- The electron and positron annihilate producing two photons, each of energy 0.511 MeV.
- The precise photon energies produced in the annihilation can be used to calibrate detectors. The same principle is used in *Positron Emission Tomography* (PET-scans).

Electromagnetic Showers

- Electrons in materials radiate photons.
- Photons of sufficient energy produce pairs.
- Each member of the pair can radiate additional photons, …
- The result is an **electromagnetic shower**.



From: http://en.wikipedia.org/wiki/Image:Shower.jpg

Cross Section and Absorption Coefficient

- The three processes are independent:

$$\mu = \mu_{\text{pe}} + \mu_{\text{Comp}} + \mu_{\text{pair}}$$

- We have two ways to express the attenuation of a beam of particles in matter:

$$\text{fraction of particles scattered out of beam} = \frac{dN}{N} = \left(\frac{A_0 \rho}{A}\right)\sigma dx = n\sigma dx$$

$$\text{fraction of beam attenuated} = \frac{dI}{I} = -\mu dx$$

- Equating these gives:

$$-\frac{dI}{I} = \frac{dN}{N} \Rightarrow \mu = n\sigma$$

Interaction of Neutrons

- Neutrons are uncharged and so do not experience Coulomb interactions.
- Slow neutrons can
  - Scatter inelastically or be captured by nuclei: subsequent decay photons can be detected.
  - Scatter elastically: nuclear recoil can produce ionization and leave a signal.
- Fast neutrons can be a source of significant background for accelerators and reactors ("albedo"). They require significant shielding structures:
  - Hydrogen-rich material to moderate.
  - Material with high neutron absorption cross section (often boron) to capture the resulting slow neutrons:

$$^{10}\mathrm{B} + n \rightarrow {}^{7}\mathrm{Li} + \alpha$$

Interaction of Hadrons at High Energies

- Hadrons: interact via the strong force.
  - **Mesons**: quark-antiquark pairs (e.g. π and K mesons)
  - **Baryons**: three quark objects (protons, neutrons, …)
- At low energies, hadronic cross sections vary rapidly with energy as various resonances and production channels "open".
- At high energies (beyond 5 GeV), hadron-hadron cross sections drop slowly with energy reaching minimum values (typically 20-40 mb) at ~70-100 GeV and then increase logarithmically with energy.

Detecting Hadrons

- Hadrons interacting with matter can produce various particles and cause nuclear excitations and breakup.
- Through many such interactions hadrons deposit energy, mostly along their incident direction, in materials.
- The resulting energy can be detected. This is the operating principle behind **calorimeters**, used to determine the energy of the incident hadron.
- Hadrons and electrons have markedly differing rates of energy loss as they traverse materials. Differences in development of the *electromagnetic shower* or *hadronic shower* can be used as a means of particle identification.