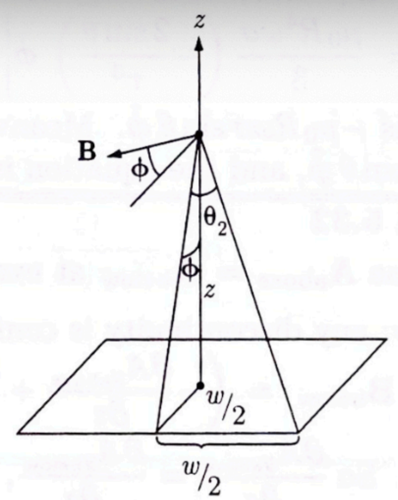
Problem 5.24

$$A_\phi = k \Rightarrow \mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{s}\frac{\partial}{\partial s}(sk)\,\hat{\mathbf{z}} = \frac{k}{s}\,\hat{\mathbf{z}};\ \mathbf{J} = \frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}\left(\frac{k}{s}\right)\right]\hat{\boldsymbol{\phi}} = \boxed{\frac{k}{\mu_0 s^2}\,\hat{\boldsymbol{\phi}}.}$$

Problem 5.36



The field of one side is given by Eq. 5.37, with $s \to \sqrt{z^2 + (w/2)^2}$ and $\sin\theta_2 = -\sin\theta_1 = \dfrac{(w/2)}{\sqrt{z^2 + w^2/2}}$;

$B = \dfrac{\mu_0 I}{4\pi}\dfrac{w}{\sqrt{z^2 + (w^2/4)}\sqrt{z^2 + (w^2/2)}}$. To pick off the vertical component, multiply by $\sin\phi = \dfrac{(w/2)}{\sqrt{z^2 + (w/2)^2}}$; for all four sides, multiply by 4: $\boxed{\mathbf{B} = \dfrac{\mu_0 I}{2\pi}\dfrac{w^2}{(z^2 + w^2/4)\sqrt{z^2 + w^2/2}}\,\hat{\mathbf{z}}.}$

For $z \gg w$, $\mathbf{B} \approx \dfrac{\mu_0 I w^2}{2\pi z^3}\,\hat{\mathbf{z}}$. The field of a dipole $\boxed{m = Iw^2,}$ for points on the z axis (Eq. 5.88, with $r \to z$, $\hat{\mathbf{r}} \to \hat{\mathbf{z}}$, $\theta = 0$) is $\mathbf{B} = \dfrac{\mu_0}{2\pi}\dfrac{m}{z^3}\,\hat{\mathbf{z}}$. ✓

Problem 5.44

From Eq. 5.24, $\mathbf{F} = \int (\mathbf{K} \times \mathbf{B}_{\text{ave}})\,da$. Here $\mathbf{K} = \sigma\mathbf{v}$, $\mathbf{v} = \omega R \sin\theta\,\hat{\boldsymbol{\phi}}$, $da = R^2 \sin\theta\,d\theta\,d\phi$, and $\mathbf{B}_{\text{ave}} = \frac{1}{2}(\mathbf{B}_{\text{in}} + \mathbf{B}_{\text{out}})$. From Eq. 5.70,

$$\mathbf{B}_{\text{in}} = \frac{2}{3}\mu_0\sigma R\omega\,\hat{\mathbf{z}} = \frac{2}{3}\mu_0\sigma R\omega(\cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}).\text{ From Eq. 5.69,}$$

$$\mathbf{B}_{\text{out}} = \nabla \times \mathbf{A} = \nabla \times \left(\frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\,\hat{\boldsymbol{\phi}}\right) = \frac{\mu_0 R^4\omega\sigma}{3}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r^2}\right)\hat{\mathbf{r}} - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\sin\theta}{r}\right)\hat{\boldsymbol{\theta}}\right]$$

$$= \frac{\mu_0 R^4\omega\sigma}{3r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}) = \frac{\mu_0 R\omega\sigma}{3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})\ \text{(since } r = R\text{)}.$$

$$\mathbf{B}_{\text{ave}} = \frac{\mu_0 R\omega\sigma}{6}(4\cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}).$$

$$\mathbf{K} \times \mathbf{B}_{\text{ave}} = (\sigma\omega R\sin\theta)\left(\frac{\mu_0 R\omega\sigma}{6}\right)\left[\hat{\boldsymbol{\phi}} \times (4\cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})\right] = \frac{\mu_0}{6}(\sigma\omega R)^2(4\cos\theta\,\hat{\boldsymbol{\theta}} + \sin\theta\,\hat{\mathbf{r}})\sin\theta.$$

Picking out the z component of $\hat{\boldsymbol{\theta}}$ (namely, $-\sin\theta$) and of $\hat{\mathbf{r}}$ (namely, $\cos\theta$), we have $(\mathbf{K} \times \mathbf{B}_{\text{ave}})_z = -\dfrac{\mu_0}{2}(\sigma\omega R)^2\sin^2\theta\cos\theta$, so

$$F_z = -\frac{\mu_0}{2}(\sigma\omega R)^2 R^2 \int \sin^3\theta\cos\theta\,d\theta\,d\phi = -\frac{\mu_0}{2}(\sigma\omega R^2)^2\,2\pi\left(\frac{\sin^4\theta}{4}\right)\Bigg|_0^{\pi/2},\ \text{or}\ \boxed{\mathbf{F} = -\frac{\mu_0\pi}{4}(\sigma\omega R^2)^2\,\hat{\mathbf{z}}.}$$