Problem 6.17

From Eq. 6.20: $\oint \mathbf{H}\cdot d\mathbf{l} = H(2\pi s) = I_{f_{\text{enc}}} = \begin{cases} I(s^2/a^2), & (s<a); \\ I & (s>a). \end{cases}$

$$H = \left\{\begin{array}{ll} \frac{Is}{2\pi a^2}, & (s<a) \\ \frac{I}{2\pi s}, & (s>a) \end{array}\right\}, \quad \text{so } B = \mu H = \boxed{\begin{cases} \frac{\mu_0(1+\chi_m)Is}{2\pi a^2}, & (s<a); \\ \frac{\mu_0 I}{2\pi s}, & (s>a). \end{cases}}$$

$\mathbf{J}_b = \chi_m \mathbf{J}_f$ (Eq. 6.33), and $J_f = \frac{I}{\pi a^2}$, so $\boxed{J_b = \frac{\chi_m I}{\pi a^2}}$ (same direction as I).

$\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}} = \chi_m \mathbf{H}\times\hat{\mathbf{n}} \Rightarrow \boxed{\mathbf{K}_b = \frac{\chi_m I}{2\pi a}}$ (opposite direction to I).

$I_b = J_b(\pi a^2) + K_b(2\pi a) = \chi_m I - \chi_m I = \boxed{0}$ (as it *should* be, of course).

Problem 6.24

(a) Forces on the upper charge:

$$\mathbf{F}_q = \frac{1}{4\pi\epsilon_0}\frac{q^2}{z^2}\hat{\mathbf{z}}, \quad \mathbf{F}_m = \nabla(\mathbf{m}\cdot\mathbf{B}) = \nabla\left(m\frac{2\mu_0 m}{4\pi z^3}\right) = \frac{\mu_0 m^2}{2\pi}\left(\frac{-3}{z^4}\right)\hat{\mathbf{z}}.$$

At equilibrium,

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{z^2} = \frac{3\mu_0 m^2}{2\pi z^4} \Rightarrow z^2 = \frac{6\mu_0\epsilon_0 m^2}{q^2} \Rightarrow z = \boxed{\sqrt{6}\frac{m}{qc},}$$

where $1/\sqrt{\epsilon_0\mu_0} = c$, the speed of light.

(b) For electrons, $q = 1.6\times 10^{-19}$ C (actually, it's the *magnitude* of the charge we want in the expression above), and $m = 9.22\times 10^{-24}$ A m^2 (the Bohr magneton—see Problem 5.58), so

$$z = \sqrt{6}\frac{9.22\times 10^{-24}}{(1.6\times 10^{-19})(3\times 10^{8})} = \boxed{4.72\times 10^{-13}\,\text{m.}}$$

(For comparison, the Bohr radius is 0.5×10^{-10} m, so the equilibrium separation is about 1% of the size of a hydrogen atom.)

(c) Good question! Certainly the answer is $\boxed{\text{no.}}$ Presumably this is an unstable equilibrium, so unless you could find a way to maintain the orientation of the dipoles, and keep them on the z axis, the structure would

Problem 6.27

At the interface, the perpendicular component of $\mathbf{B}$ is continuous (Eq. 6.26), and the parallel component of $\mathbf{H}$ is continuous (Eq. 6.25 with $\mathbf{K}_f = 0$). So $B_1^\perp = B_2^\perp$, $\mathbf{H}_1^\parallel = \mathbf{H}_2^\parallel$. But $\mathbf{B} = \mu\mathbf{H}$ (Eq. 6.31), so $\frac{1}{\mu_1}\mathbf{B}_1^\parallel = \frac{1}{\mu_2}\mathbf{B}_2^\parallel$. Now $\tan\theta_1 = B_1^\parallel/B_1^\perp$, and $\tan\theta_2 = B_2^\parallel/B_2^\perp$, so

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{B_2^\parallel}{B_2^\perp}\frac{B_1^\perp}{B_1^\parallel} = \frac{B_2^\parallel}{B_1^\parallel} = \frac{\mu_2}{\mu_1}$$

(the same form, though for different reasons, as Eq. 4.68).