Problem 2.54.
For point charge at the center
(a)

$$\vec{E} \;=\; \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\big(1+\frac{r}{\lambda}\big)e^{-\frac{r}{\lambda}}$$

⇒ for a continuous charge distribution

$$\vec{E} \;=\; \frac{1}{4\pi\epsilon_0}\int d^3x'\;\frac{\rho\hat{\varsigma}}{\varsigma^2}\big(1+\frac{\varsigma}{\lambda}\big)e^{-\frac{\varsigma}{\lambda}}\Big|_{\vec{\varsigma}=\vec{r}-\vec{r}'}$$

(b)

$$\vec{\nabla}\times\vec{E} \;=\; 0 \qquad \Rightarrow \qquad \vec{E}(\vec{r}) \;=\; -\,\vec{\nabla}V(\vec{r})$$

(c)
For a point charge at the origin

$$V(r) \;=\; \frac{1}{4\pi\epsilon_0}\int_r^\infty \vec{E}\cdot d\vec{l} \;=\; \frac{1}{4\pi\epsilon_0}\int_r^\infty dr'\Big[\frac{1}{r'^2}e^{-\frac{r'}{\lambda}}+\frac{1}{\lambda r'}e^{-\frac{r'}{\lambda}}\Big] \;\overset{\text{by parts}}{=}\; \frac{q}{4\pi\epsilon_0 r}e^{-\frac{r}{\lambda}}$$

⇒ for a continuous charge distribution

$$V(\vec{r}) \;=\; \frac{1}{4\pi\epsilon_0}\int d^3x'\rho(\vec{r}')\frac{1}{\varsigma}e^{-\frac{\varsigma}{\lambda}}\Big|_{\vec{\varsigma}=\vec{r}-\vec{r}'}$$

(d+e)

$$\begin{aligned}
\vec{\nabla}\cdot\vec{E}(\vec{r}) \;&=\; \frac{1}{4\pi\epsilon_0}\int d^3x\rho(\vec{r}')\;\vec{\nabla}_r\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\big(1+\frac{|\vec{r}-\vec{r}'|}{\lambda}\big)e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda}}\\
&=\; \frac{1}{4\pi\epsilon_0}\int d^3x'\rho(\vec{r}')\;\Big\{\big(1+\frac{|\vec{r}-\vec{r}'|}{\lambda}\big)e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda}}\vec{\nabla}_r\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}+\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\cdot\vec{\nabla}_r\big(1+\frac{|\vec{r}-\vec{r}'|}{\lambda}\big)e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda}}\Big\}\\
&=\; \frac{1}{4\pi\epsilon_0}\int d^3x'\rho(\vec{r}')\;\Big\{\big(1+\frac{|\vec{r}-\vec{r}'|}{\lambda}\big)e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda}}4\pi\delta^{(3)}(\vec{r}-\vec{r}')+\frac{\hat{\varsigma}}{\varsigma^2}\cdot\hat{\varsigma}\frac{\partial}{\partial\varsigma}\Big(1+\frac{\varsigma}{\lambda}\Big)e^{-\frac{\varsigma}{\lambda}}\Big|_{\vec{\varsigma}=\vec{r}-\vec{r}'}\Big\}\\
&=\; \frac{\rho(\vec{r})}{\epsilon_0}-\frac{1}{\lambda^2}\frac{1}{4\pi\epsilon_0}\int d^3x'\rho(\vec{r}')\frac{1}{\varsigma}e^{-\frac{\varsigma}{\lambda}}\Big|_{\vec{\varsigma}=\vec{r}-\vec{r}'} \;=\; \frac{\rho(\vec{r})}{\epsilon_0}-\frac{1}{\lambda^2}V(\vec{r})
\end{aligned}$$

Due to Gauss theorem

$$\int_{S=\partial\mathcal{V}}\vec{E}\cdot d\vec{a} \;=\; \int_{\mathcal{V}}d^3x'\;\vec{\nabla}\cdot\vec{E}(\vec{r}) \;=\; \int_{\mathcal{V}}d^3x'\;\Big[\frac{\rho(\vec{r}')}{\epsilon_0}-\frac{1}{\lambda^2}V(\vec{r}')\Big] \;=\; \frac{Q_{\text{enc}}}{\epsilon_0}-\frac{1}{\lambda^2}\int_{\mathcal{V}}d^3x'\;V(\vec{r}')$$