Problem 5.7

$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}\int_{\mathcal{V}} \rho\mathbf{r}\,d\tau = \int\left(\frac{\partial\rho}{\partial t}\right)\mathbf{r}\,d\tau = -\int(\boldsymbol{\nabla}\cdot\mathbf{J})\mathbf{r}\,d\tau$ (by the continuity equation). Now product rule #5 says $\boldsymbol{\nabla}\cdot(x\mathbf{J}) = x(\boldsymbol{\nabla}\cdot\mathbf{J}) + \mathbf{J}\cdot(\boldsymbol{\nabla}x)$. But $\boldsymbol{\nabla}x = \hat{\mathbf{x}}$, so $\boldsymbol{\nabla}\cdot(x\mathbf{J}) = x(\boldsymbol{\nabla}\cdot\mathbf{J}) + J_x$. Thus $\int_{\mathcal{V}}(\boldsymbol{\nabla}\cdot\mathbf{J})x\,d\tau = \int_{\mathcal{V}}\boldsymbol{\nabla}\cdot(x\mathbf{J})\,d\tau - \int_{\mathcal{V}} J_x\,d\tau$. The first term is $\int_{\mathcal{S}} x\mathbf{J}\cdot d\mathbf{a}$ (by the divergence theorem), and since $\mathbf{J}$ is entirely *inside* $\mathcal{V}$, it is zero on the surface $\mathcal{S}$. Therefore $\int_{\mathcal{V}}(\boldsymbol{\nabla}\cdot\mathbf{J})x\,d\tau = -\int_{\mathcal{V}} J_x\,d\tau$, or, combining this with the y and z components, $\int_{\mathcal{V}}(\boldsymbol{\nabla}\cdot\mathbf{J})\mathbf{r}\,d\tau = -\int_{\mathcal{V}}\mathbf{J}\,d\tau$. Or, referring back to the first line, $\frac{d\mathbf{p}}{dt} = \int\mathbf{J}\,d\tau$. qed

Here's a quicker method, if the distribution consists of a collection of point charges. Use Eqs. 5.30 and 3.100:

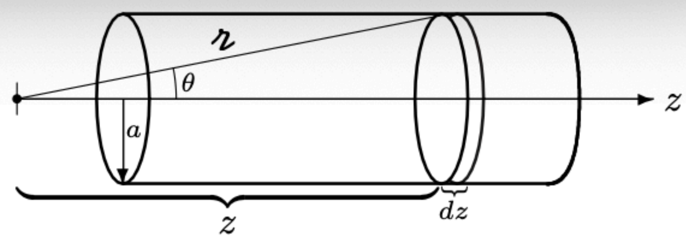
$$\int\mathbf{J}\,d\tau = \sum q_i\mathbf{v}_i = \frac{d}{dt}\sum q_i\mathbf{r}_i = \frac{d\mathbf{p}}{dt}.$$

Problem 5.11

Use Eq. 5.41 for a ring of width dz, with $I \to nI\,dz$:

$B = \frac{\mu_0 nI}{2}\int\frac{a^2}{(a^2+z^2)^{3/2}}\,dz$. But $z = a\cot\theta$,

so $dz = -\frac{a}{\sin^2\theta}\,d\theta$, and $\frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$.



So

$$B = \frac{\mu_0 nI}{2}\int\frac{a^2\sin^3\theta}{a^3\sin^2\theta}(-a\,d\theta) = -\frac{\mu_0 nI}{2}\int\sin\theta\,d\theta = \frac{\mu_0 nI}{2}\cos\theta\Big|_{\theta_1}^{\theta_2} = \boxed{\frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1).}$$

For an infinite solenoid, $\theta_2 = 0$, $\theta_1 = \pi$, so $(\cos\theta_2 - \cos\theta_1) = 1 - (-1) = 2$, and $B = \boxed{\mu_0 nI.}$ ✓

Problem 5.13

Magnetic attraction per unit length (Eqs. 5.40 and 5.13): $f_m = \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d}$.

Electric field of one wire (Eq. 2.9): $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}$. Electric repulsion per unit length on the other wire: $f_e = \frac{1}{2\pi\epsilon_0}\frac{\lambda^2}{d}$. They balance when $\mu_0 v^2 = \frac{1}{\epsilon_0}$, or $\boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}}.}$ Putting in the numbers,