Problem 2 solution

(a) The energy goes into increasing internal energy⇔termperature of the gas.
(b) Change of entropy ($x \equiv \delta$)

$$S(x) \;=\; \frac{3}{2}kN\ln(U_0 + fx) + kN\ln(V_0 - Ax) + \text{const} \quad\Rightarrow\quad \Delta S \;=\; \frac{3}{2}kN\ln\frac{U_0 + fx}{U_0} + kN\ln\frac{V_0 - Ax}{V_0}$$

(b) Equilibrium position corresponds to maximal entropy

$$\frac{d}{dx}\Delta S(x) \;=\; \frac{3}{2}fkN\frac{1}{U_0 + fx} - kNA\frac{1}{V_0 - Ax} \;=\; 0 \quad\Rightarrow\quad x_* \;=\; \frac{3}{5}L - \frac{2}{5}\frac{U_0}{f}$$

Check that it is a max and not a min

$$\frac{d^2}{dx^2}\Delta S(x) \;=\; -\frac{3}{2}f^2kN\frac{1}{(U_0 + fx)^2} + kNA^2\frac{1}{(V_0 - Ax)^2}\Big|_{x=x_*} \;=\; \frac{3}{4}f^2kN\frac{1}{(U_0 + fx)^2}$$

(c) For an ideal (monoatomic) gas $PV \;=\; NkT \;=\; \frac{2}{3}U$. In the equilibrium $P_*A = f$ and $U = U_0 + fx$ so

$$P_*V_* = \; f(L - x_*) = \frac{2}{3}(U_0 + fx_*) \quad\Rightarrow\quad x_* \;=\; \frac{3}{5}L - \frac{2}{5}\frac{U_0}{f}$$