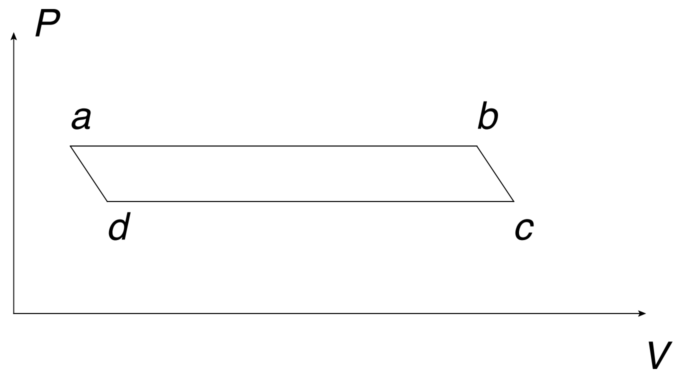
Problem 8-20 solution

(a)



(b)

$$W = P_1(V_b - V_a) + P_2(V_d - V_c) \simeq dP(V_b - V_a) = \frac{du}{3}(V_b - V_a)$$

(c)

$$Q_{\text{in}} = U_b - U_a + W_{ab} = u(V_b - V_a) + \frac{u}{3}(V_b - V_a) = \frac{4u}{3}(V_b - V_a)$$

(d)

$$\eta = \frac{W}{Q_{\text{in}}} = \frac{du}{4u}$$

For Carnot cycle $\eta = 1 - \frac{T-dT}{T} = \frac{dT}{T}$ so

$$\frac{du}{4u} = \frac{dT}{T} \Rightarrow u = \text{const} \times T^4$$

Problem 8-23 solution

From Eq. (8.6)

$$dU = TdS + \mathcal{H}dM \Rightarrow dU(T,\mathcal{H}) = T\frac{\partial S(T,\mathcal{H})}{\partial T}dT + T\frac{\partial S(T,H)}{\partial \mathcal{H}}d\mathcal{H} + \mathcal{H}\frac{\partial M(T,\mathcal{H})}{\partial T}dT + \mathcal{H}\frac{\partial M(T,\mathcal{H})}{\partial \mathcal{H}}d\mathcal{H}$$

$$\Rightarrow \frac{\partial U(T,\mathcal{H})}{\partial \mathcal{H}} = T\frac{\partial S(T,\mathcal{H})}{\partial \mathcal{H}} + \mathcal{H}\frac{\partial M(T,\mathcal{H})}{\partial \mathcal{H}} \Leftrightarrow \left(\frac{\partial U}{\partial \mathcal{H}}\right)_T = T\left(\frac{\partial S}{\partial \mathcal{H}}\right)_T + \mathcal{H}\left(\frac{\partial M}{\partial \mathcal{H}}\right)_T$$

From Eq. (8-75) $\left(\frac{\partial S}{\partial \mathcal{H}}\right)_T = \left(\frac{\partial M}{\partial T}\right)_{\mathcal{H}}$ so we get

$$\left(\frac{\partial U}{\partial \mathcal{H}}\right)_T = T\left(\frac{\partial M}{\partial T}\right)_{\mathcal{H}} + \mathcal{H}\left(\frac{\partial M}{\partial \mathcal{H}}\right)_T$$

Due to Curie's law $M = C_c\frac{\mathcal{H}}{T}$

$$\mathcal{H}\left(\frac{\partial M}{\partial \mathcal{H}}\right)_T = -T\left(\frac{\partial M}{\partial T}\right)_{\mathcal{H}} = M \quad \Rightarrow \quad \left(\frac{\partial U}{\partial \mathcal{H}}\right)_T = 0 \quad \Rightarrow \quad U = U(T)$$

Problem 9-7 solution

(b):

$$N \;=\; \int_0^{v_0} dv\;\frac{dN}{dv} \;=\; k\int_0^{v_0} dv\; v \;=\; k\frac{v_0^2}{2} \quad\Rightarrow\quad k \;=\; \frac{2N}{v_0^2}$$

(c):

$$\bar{v} \;=\; \frac{1}{N}\int_0^{v_0} dv\; v\frac{dN}{dv} \;=\; \frac{1}{N}k\int_0^{v_0} dv\; v^2 \;=\; \frac{kv_0^3}{3N} \;=\; \frac{2}{3}v_0$$

(d):

$$\overline{v^2} \;=\; \frac{1}{N}\int_0^{v_0} dv\; v^2\frac{dN}{dv} \;=\; \frac{k}{N}\int_0^{v_0} dv\; v^3 \;=\; \frac{kv_0^4}{4N} \;=\; \frac{v_0^2}{2} \quad\Rightarrow\quad \sqrt{\overline{v^2}} \;=\; \frac{v_0}{\sqrt{2}}$$