HW 9 solutions

Problem 9-18 solution

From Eq. (9.11)

$$\Phi_{\mathrm{l}\to\mathrm{r}} \;=\; \bar{v}\frac{n_{\mathrm{l}}}{4}, \quad \Phi_{\mathrm{r}\to\mathrm{l}} \;=\; \bar{v}\frac{n_{\mathrm{r}}}{4} \qquad \Rightarrow \; \frac{dN_{\mathrm{l}}}{dt} \;=\; A(\Phi_{\mathrm{r}\to\mathrm{l}} - \Phi_{\mathrm{l}\to\mathrm{r}}) \;=\; \frac{A\bar{v}}{4}(n_{\mathrm{r}} - n_{\mathrm{l}}) \;=\; \frac{A\bar{v}}{4}(n_0 - 2n_{\mathrm{l}})$$

Since $\bar{v}$ and $\overline{v^2}$ does not depend on time (they depend only on temperature which is kept the same in both volumes), we get the differential equation

$$\frac{dN_{\mathrm{l}}(t)}{dt} \;=\; \frac{A\bar{v}}{4}[n_0 - 2n_{\mathrm{l}}(t)] \quad \Rightarrow \quad \frac{dn_{\mathrm{l}}(t)}{dt} \;=\; \frac{A\bar{v}}{4V}[n_0 - 2n_{\mathrm{l}}(t)]$$

with initial condition $n_{\mathrm{l}}\Big|_{t=0} = n_0$.
Solution:

$$\frac{dn_{\mathrm{l}}}{n_0 - 2n_{\mathrm{l}}} \;=\; \frac{A\bar{v}}{4V}dt \;\;\Rightarrow\;\; \ln(n_0 - 2n_{\mathrm{l}}) \;=\; -\frac{A\bar{v}}{2V}t \;+\; \text{const} \;\;\Rightarrow\;\; n_0 - 2n_{\mathrm{l}} \;=\; C\times e^{-\frac{A\bar{v}}{2V}t}$$

From the initial condition we get $C = -n_0$ so the solution is

$$n_{\mathrm{l}}(t) \;=\; \frac{n_0}{2}\left(1 + e^{-\frac{A\bar{v}}{2}t}\right)$$

As expected, at $t \to \infty$ we have $n_{\mathrm{l}} = n_{\mathrm{r}} = \frac{n_0}{2}$

In terms of pressure $P = \frac{nm\overline{v^2}}{3}$ we get

$$P \;=\; \frac{P_0}{2}\left(1 + e^{-\frac{A\bar{v}}{2V}t}\right)$$

Problem 9-20 solution

(a):

In the adiabatic process

$$\delta W \;=\; PdV \;=\; NkT\frac{dV}{V} \;=\; -dU \;=\; -\frac{3}{2}NkdT \;\;\Rightarrow\;\; \frac{dV}{V} \;=\; -\frac{3}{2}\frac{dT}{T} \;\;\Rightarrow\;\; V \;=\; \text{const}\times T^{-\frac{3}{2}} \;\;\Rightarrow\;\; VT^{\frac{3}{2}} \;=\; \text{const}$$

Since $PV \;=\; NkT$ we get

$$V(PV)^{\frac{3}{2}} \;=\; \text{const} \;\;\Rightarrow\;\; P^{\frac{3}{2}}V^{\frac{5}{2}} \;=\; \text{const} \;\;\Leftrightarrow\;\; PV^{\frac{5}{3}} \;=\; \text{const}$$

(b):
$TP^{-\frac{2}{5}} \;=\; \text{const} \;\;\Rightarrow\;\; T \;\sim\; P^{\frac{2}{5}}$

$$\Rightarrow \;\; \overline{v^2} \;\sim\; T \;\;\Rightarrow\;\; \sqrt{\overline{v^2}} \;\sim\; \sqrt{T} \;\sim\; P^{\frac{1}{5}}$$