

HW 9 solutions

Problem 9-7 solution

(b):

$$N = \int_0^{v_0} dv \frac{dN}{dv} = k \int_0^{v_0} dv v = k \frac{v_0^2}{2} \Rightarrow k = \frac{2N}{v_0^2}$$

(c):

$$\bar{v} = \frac{1}{N} \int_0^{v_0} dv v \frac{dN}{dv} = \frac{1}{N} k \int_0^{v_0} dv v^2 = \frac{kv_0^3}{3N} = \frac{2}{3}v_0$$

(d):

$$\overline{v^2} = \frac{1}{N} \int_0^{v_0} dv v^2 \frac{dN}{dv} = \frac{k}{N} \int_0^{v_0} dv v^3 = \frac{kv_0^4}{4N} = \frac{v_0^2}{2} \Rightarrow \sqrt{\overline{v^2}} = \frac{v_0}{\sqrt{2}}$$

Problem 9-18 solution

From Eq. (9.11)

$$\Phi_{1 \rightarrow r} = \bar{v} \frac{n_1}{4}, \quad \Phi_{r \rightarrow 1} = \bar{v} \frac{n_r}{4} \Rightarrow \frac{dN_1}{dt} = A(\Phi_{r \rightarrow 1} - \Phi_{1 \rightarrow r}) = \frac{A\bar{v}}{4}(n_r - n_1) = \frac{A\bar{v}}{4}(n_0 - 2n_1)$$

Since \bar{v} and $\overline{v^2}$ does not depend on time (they depend only on temperature which is kept the same in both volumes), we get the differential equation

$$\frac{dN_1(t)}{dt} = \frac{A\bar{v}}{4}[n_0 - 2n_1(t)] \Rightarrow \frac{dn_1(t)}{dt} = \frac{A\bar{v}}{4V}[n_0 - 2n_1(t)]$$

with initial condition $n_1 \Big|_{t=0} = n_0$.

Solution:

$$\frac{dn_1}{n_0 - 2n_1} = \frac{A\bar{v}}{4V} dt \Rightarrow \ln(n_0 - 2n_1) = -\frac{A\bar{v}}{2V}t + \text{const} \Rightarrow n_0 - 2n_1 = C \times e^{-\frac{A\bar{v}}{2V}t}$$

From the initial condition we get $C = -n_0$ so the solution is

$$n_1(t) = \frac{n_0}{2}(1 + e^{-\frac{A\bar{v}}{2V}t})$$

As expected, at $t \rightarrow \infty$ we have $n_1 = n_r = \frac{n_0}{2}$

In terms of pressure $P = \frac{nm\overline{v^2}}{3}$ we get

$$P = \frac{P_0}{2}(1 + e^{-\frac{A\bar{v}}{2V}t})$$

Problem 9-20 solution

(a):

In the adiabatic process

$$\delta W = PdV = NkT \frac{dV}{V} = -dU = -\frac{3}{2}NkdT \Rightarrow \frac{dV}{V} = -\frac{3}{2} \frac{dT}{T} \Rightarrow V = \text{const} \times T^{-\frac{3}{2}} \Rightarrow VT^{\frac{3}{2}} = \text{const}$$

Since $PV = NkT$ we get

$$V(PV)^{\frac{3}{2}} = \text{const} \Rightarrow P^{\frac{3}{2}}V^{\frac{5}{2}} = \text{const} \Leftrightarrow PV^{\frac{5}{3}} = \text{const}$$

(b):

$$TP^{-\frac{2}{5}} = \text{const} \Rightarrow T \sim P^{\frac{2}{5}}$$

$$\Rightarrow \overline{v^2} \sim T \Rightarrow \sqrt{\overline{v^2}} \sim \sqrt{T} \sim P^{\frac{1}{5}}$$