Problem 1.

You store 10.0 kg of ice at -20.0 deg C in a perfectly thermally insulating vessel. You then open it, quickly pour 2.0 kg of water at +40.0 deg C on the ice, and immediately seal the vessel.

(a) What will the temperature be when the system has reached thermal equilibrium? [Ignore thermal effects of air.]

(b)How much ice (in kg) will there be in the end?

Specific heat of water: 4184 J/(kg K)

Specific heat of ice: 2060 J/(kg K)

Latent heat of fusion of water: 334.2 kJ/kg

Solution.

We have 3 possibilities: all ice melt, all water freeze, or ice and water coexist at 0°C.

Consider them in turn:

1. Maximal amount of heat released due to cooling of the water is $4184 J/(kgK) \times 40°K \times 2kg = $ 334.72kJ which is smaller than the heat $334.2 \times 10kJ = 3342kJ$ necessary to melt all ice.

2. Maximal amount of heat absorbed due to warming of the ice is $2060 J/(kgK) \times 20°K \times 10kg =$ 412kJ which is smaller than the heat $334.2(kJ/kg) \times 2kg =$ 668.4kJ necessary to freeze all water.

3. Thus, the final temperature is 0°C and some water has freezed (or some ice melted). Denote mass of the new ice m_x (it can be negative). We get

$$
\begin{aligned}
c_1 m_1 \Delta T_1 - c_2 m_2 \Delta T_2 &= l m_x \\
\Rightarrow\ m_x = \frac{c_1 m_1 \Delta T_1 - c_2 m_2 \Delta T_2}{l} &= \frac{2.06 \times 10 \times 20 - 4.184 \times 2 \times 40}{334.2} = \frac{412 - 334.72}{334.2} \simeq 0.23\text{kg}
\end{aligned}
$$

Overall, there will be 10.23 kg of ice and 1.77 kg of water at 0°C.

Problem 2.

A heat engine operates on an ideal monoatomic gas in the reversible cycle consisting of isochoric, isobaric and two isothermal processes as shown below. What is the efficiency of this heat engine? Compare it to Carnot efficiency $1 - \frac{T_1}{T_2}$.

Solution.

The internal energy for an ideal gas is $u = c_v T$, and for the monoatomic gas $c_v = \frac{3}{2}R$.

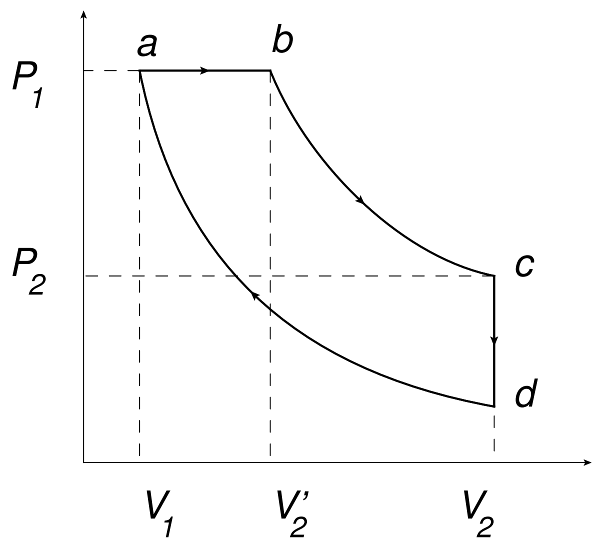

FIG. 1. PV diagram for a heat engine. Curved lines are isotherms.

Let us assume positive sign for heat going into the device and consider four processes of the cycle.

1. Process a → b.

$$P_1 V_2' = P_2 V_2 \quad \Rightarrow \quad W_{ab} \; = \; P_1(V_2' - V_1) \; = \; P_2 V_2 - P_1 V_1 \; = \; nR(T_2 - T_1)$$

$$U_b - U_a \; = \; C_v(T_2 - T_1) \quad \Rightarrow \quad Q_{ab} \; = \; n(c_v + R)(T_2 - T_1) \; = \; n\frac{5R}{2}(T_2 - T_1)$$

2. Process b → c.

$$U_b = U_c \quad \Rightarrow \quad Q_{bc} \; = \; W_{bc} \; = \; nRT_2 \ln \frac{V_2}{V_2'} \; = \; nRT_2 \ln \frac{P_1}{P_2}$$

3. Process c→d.

$$W_{cd} \; = \; 0 \quad \Rightarrow \quad Q_{cd} \; = \; U_d - U_c \; = \; nc_v(T_1 - T_2)$$

4. Process d→a.

$$U_d = U_a \quad \Rightarrow \quad Q_{da} \; = \; W_{da} \; = \; nRT_1 \ln \frac{V_1}{V_2}$$

Total work:

$$W \; = \; W_{ab} + W_{bc} + W_{da} \; = \; nR(T_2 - T_1) + nRT_2 \ln \frac{P_1}{P_2} - nRT_1 \ln \frac{V_2}{V_1}$$

Total heat absorbed:

$$Q_{ab} + Q_{bc} \; = \; \frac{5}{2} nR(T_2 - T_1) + nRT_2 \ln \frac{P_1}{P_2}$$

Check: total heat rejected $|Q_{cd}| + |Q_{da}| \; = \; \frac{3}{2}nR(T_2 - T_1) + nRT_1 \ln\frac{V_2}{V_1}$ so

$$Q_{ab} + Q_{bc} - |Q_{cd}| - |Q_{da}| \; = \; W \qquad \text{OK}$$

Efficiency:

$$\eta \; = \; \frac{W}{Q_{ab} + Q_{bc}} \; = \; \frac{nR(T_2 - T_1) + nRT_2 \ln\frac{P_1}{P_2} - nRT_1 \ln\frac{V_2}{V_1}}{\frac{5}{2}nR(T_2 - T_1) + nRT_2 \ln\frac{P_1}{P_2}} \; = \; \frac{T_2 - T_1 + T_2 \ln\frac{P_1}{P_2} - T_1 \ln\frac{V_2}{V_1}}{\frac{5}{2}(T_2 - T_1) + T_2 \ln\frac{P_1}{P_2}}$$

Compare to Carnot efficiency

$$\frac{T_2 - T_1 + T_2 \ln\frac{P_1}{P_2} - T_1 \ln\frac{V_2}{V_1}}{\frac{5}{2}(T_2 - T_1) + T_2 \ln\frac{P_1}{P_2}} \; < \; 1 - \frac{T_1}{T_2}$$

since

$$\frac{5}{2}\frac{(T_2 - T_1)^2}{T_2} + T_1 \ln\frac{P_2 V_2}{P_1 V_1} \; = \; \frac{5}{2}\frac{(T_2 - T_1)^2}{T_2} + T_1 \ln\frac{T_2}{T_1} \; > \; T_2 - T_1$$

Problem 3.

Problem 4.27 from the textbook:

The equation of state is $P \; = \; \frac{a}{3}T^4$ and the energy equation is $U \; = \; aT^4 V$

Solution.

(a)

Denote initial volume by V_0. Change of intermal energy is

$$\Delta U \; = \; aT^4 \Delta V \; = \; aT^4 V_0$$

The work done by the system in the isothermal expansion $V_0 \to 2V_0$ is

$$W \; = \; \int P dV \; = \; \frac{a}{3}T^4 \Delta V \; = \; \frac{a}{3}T^4 V_0$$

so from the first law

$$\Delta Q \; = \; \Delta U + \Delta W \; = \; \frac{4}{3}aT^4 V_0$$

(b)

Eq. (4.3) reads

$$\delta q \; = \; \Big(\frac{\partial u}{\partial T}\Big)_v dT + \Big[\Big(\frac{\partial u}{\partial v}\Big)_T + P\Big] dv$$

In our case $\left(\frac{\partial u}{\partial T}\right)_v \; = \; 4aT^3$, $\left(\frac{\partial u}{\partial v}\right)_T \; = \; aT^4$, and $P = \frac{a}{3}T^4$. Since in the adiabatic process $\delta q \; = \; 0$, we get

$$[aT^4 + \frac{a}{3}T^4] dv + 4aT^3 dT \; = \; 0 \quad \Rightarrow \quad \Big(\frac{dv}{dT}\Big)_s \; = \; -\frac{3}{T} \quad \Rightarrow \quad V \; = \; \frac{\text{const}}{T^3}$$

Problem 4.

A kilomole of ideas gas undergoes a reversible isothermal expansion from volume V to volume $3V$.

(a) What is the change in entropy of the gas?

(b) What is the change of entropy of the universe?

Suppose now that the same expansion takes place as a free expansion. Same questions:

(c) What is the change in entropy of the gas?

(d) What is the change of entropy of the universe?

Solution.

(a):

Change in entropy of the gas in the isothermal process

$$\Delta S_{\text{gas}} = \frac{\Delta Q}{T}, \quad \Delta Q = W = nRT \ln \frac{V_2}{V_1} \Rightarrow \Delta S_{\text{gas}} = nR \ln 3$$

(b):

Change of the entropy of the surroundings

$$\Delta S_{\text{surr}} = -\frac{\Delta Q}{T}, \quad \Rightarrow \Delta S_{\text{surr}} = -nR \ln 3$$

so the entropy of the universe does not change as it should be for a reversible process.

(c):

Change of the entropy of the gas is the same as in the reversible process:

$$\Delta S_{\text{gas}} = nRT \ln \frac{V_2}{V_1} \Rightarrow \Delta S_{\text{gas}} = nR \ln 3$$

(d):

Entropy of the surroundings does not change $\Rightarrow$ the entropy of the universe is increased by $nR \ln 3$.