

A particle of mass  $m$  slides without friction around the circumference of a circular wire hoop of radius  $a$ . The hoop is placed upright in a uniform gravitational field and rotates about a vertical diameter with angular velocity  $\Omega$ .

1. Construct the Lagrangian, using as generalized coordinate the angular displacement  $\theta$  along the hoop measured from the downward vertical. Derive the differential equation for the motion and construct the corresponding first integral.
2. Using the equation of motion, obtain all positions of dynamical equilibrium and classify them as stable or unstable. For those configurations that are stable, determine the frequency of small oscillations about that position.
3. Find the Hamiltonian for the system. Is it a constant of motion? Compare to the first integral in part 1 above.