

Consider a simple, plane pendulum consisting of a mass m attached to a string of length l . After the pendulum is set into motion, the length of the string is shortened at a constant rate

$$\frac{dl}{dt} = -\alpha = \text{constant} .$$

The suspension point remains fixed.

1. Compute the Lagrangian and Hamiltonian functions.
2. Is the Hamiltonian equal to the total energy in this case ?
3. Is the mechanical energy of the mass conserved for this system ?