

Phys. 807 — Statistical Mechanics

Solution.

The effective potential is

$$V_{\text{eff}} = V(r) + \frac{mv^2}{2} \frac{b^2}{r^2}$$

To be captured, the electron must approach the distance a from the center of the ions, so its energy must be greater than the maximum of $V_{\text{eff}}(a)$

$$E = \frac{mv^2}{2} \geq V_0 + \frac{mv^2}{2} \frac{b^2}{a^2} \Rightarrow b \leq a \sqrt{1 - \frac{2V_0}{mv^2}}$$

so

$$b_{\text{max}} = a \sqrt{1 - \frac{2V_0}{mv^2}}$$

and the cross section of ionization is

$$\sigma = \pi a^2 \left(1 - \frac{2V_0}{mv^2}\right)$$