HW4 solution

Problem (a): parallel-plate capacitor

From Gauss' law

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$
$$V = Ed \;\Rightarrow\; V = \frac{Q}{A\epsilon_0}d \;\Rightarrow\; C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

Problem (b): spherical capacitor

From Gauss' law

$$E(r) = \frac{Q}{4\pi r^2\epsilon_0} \;\rightarrow\; V = \Phi(b) - \Phi(a) = -\int_a^b dr\; E(r) = \frac{Q}{4\pi\epsilon_0}(\frac{1}{a} - \frac{1}{b})$$
$$\Rightarrow\; C = \frac{Q}{V} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Problem (c): cylindrical capacitor
From Gauss' law

$$E(r) = \frac{Q}{2\pi rL\epsilon_0} \;\rightarrow\; V = \Phi(b) - \Phi(a) = -\int_a^b dr\; E(r) = \frac{Q}{2\pi L\epsilon_0}\ln\frac{b}{a}$$
$$\Rightarrow\; C = \frac{Q}{V} = \frac{2\pi L\epsilon_0}{\ln(b/a)}$$