

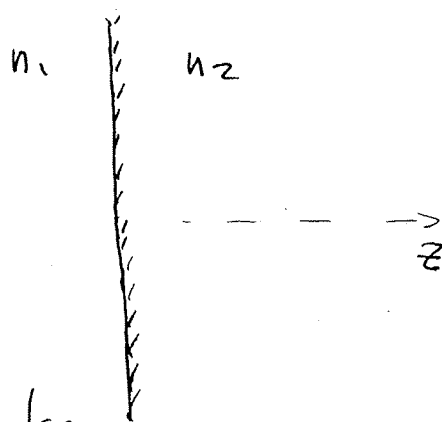
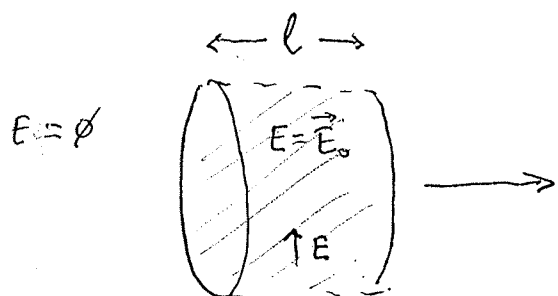
$R+T=1$  revisited

$$v_{1,2} = \frac{c}{n_{1,2}} \quad 1$$

For simplicity, consider <sup>the</sup> case of incident wave normal to the interface

$$\frac{E_0^R}{E_0^I} = \frac{n_1 - n_2}{n_1 + n_2} \quad \frac{E_0^T}{E_0^I} = \frac{2n_1}{n_1 + n_2} \quad \Rightarrow \quad R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \quad \begin{array}{l} \mu_1 = \mu_2 \\ \text{for} \\ \text{simplicity} \end{array}$$
$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

To avoid confusion due to the interference between incident and reflected waves, consider the scattering of an electromagnetic pulse



For simplicity, we take the square pulse

$$\vec{E}_I(z-v_1 t) = E_0 \hat{e}_1 s_p(z-v_1 t)$$

$$s_p(x) = \begin{cases} 1 & \text{if } 0 < x < l \\ \emptyset & \text{otherwise} \end{cases}$$

Recall that a general solution of the 1dim wave eqn  $\frac{\partial^2}{\partial z^2} f(z,t) - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} f(z,t) = 0$

is  $f_1(z-v_1 t) + f_2(z+v_1 t)$

↑  
right-moving

↓  
left-moving

How to solve the scattering problem for a pulse?

Idea: formally decompose a pulse into the sum of plane waves, solve the scattering problem for each plane wave separately, and reassemble the scattered pulse from the plane waves.

$$E_i(v_1, t-z) = \int_{-\infty}^{\infty} dk \tilde{E}_i(k) e^{-ik(v_1 t - z)} = \int_{-\infty}^{\infty} dk \tilde{E}_i(k) e^{-i\omega t + ikz}$$

Maxwell's eqns are linear  $\Rightarrow$  superposition principle  
 (if  $\vec{E}_{(1)}, \vec{B}_{(1)}$  and  $\vec{E}_{(2)}, \vec{B}_{(2)}$  are solutions of Maxwell's eqns,  
 so is  $\vec{E}_{(1)} + \vec{E}_{(2)}$  and  $\vec{B}_{(1)} + \vec{B}_{(2)}$ )  $\Rightarrow$  each  $\tilde{E}_i(k) e^{-i\omega t + ikz}$   
 is a plane wave which is transmitted (and reflected)  
 independently of other constituents with different  $k$ 's

$\Rightarrow$  solution for a certain  $k_i$  is ( $\omega = v_1 k_i$ )

$$\begin{aligned} \tilde{E}_{0i} e^{-i\omega t + ik_i z} + \tilde{E}_{0R} e^{-i\omega t + ik_R z} & \quad z < 0 & \quad k_R = -k_i \\ \tilde{E}_{0T} e^{-i\omega t + ik_T z} & \quad z > 0 & \quad k_T = \frac{\omega}{v_2} = k_i \frac{v_1}{v_2} \end{aligned}$$

$$\tilde{E}_{0R} = \frac{n_1 - n_2}{n_1 + n_2} E_{0i}, \quad \tilde{E}_{0T} = \frac{2n_1}{n_1 + n_2} E_{0i} \Rightarrow$$

$$\begin{aligned} \Rightarrow \tilde{E}_{0i} e^{-i\omega t + ik_i z} + \frac{n_1 - n_2}{n_1 + n_2} \tilde{E}_{0i} e^{-i\omega t - ik_i z} & \quad z < 0 \\ \tilde{E}_{0i} \frac{2n_1}{n_1 + n_2} e^{-i\omega t + ik_i z \frac{v_1}{v_2}} & \quad z > 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} \Rightarrow \tilde{E}_{0i} e^{-i\omega t + ik_i z} + \frac{n_1 - n_2}{n_1 + n_2} \tilde{E}_{0i} e^{-i\omega t - ik_i z} \\ \tilde{E}_{0i} \frac{2n_1}{n_1 + n_2} e^{-i\omega t + ik_i z \frac{v_1}{v_2}} \end{aligned}} \right\} \text{describes}$$

the result of the scattering of a plane wave with  $k = k_i$

$\Rightarrow$  superposition principle  $\Rightarrow$

$$E(z, t) \stackrel{z < 0}{=} \int \frac{dk}{2\pi} (\tilde{E}_i(k) e^{-ikv_1 t + ikz} + \frac{n_1 - n_2}{n_1 + n_2} \tilde{E}_i(k) e^{-i\omega_1 t - ikz})$$

$$E(z, t) \stackrel{z > 0}{=} \int \frac{dk}{2\pi} \tilde{E}_i(k) \frac{2n_1}{n_1 + n_2} e^{-ikv_1 t + ik \frac{v_1}{v_2} z}$$

Performing the integrations over  $k$ , we get

$$E(z, t) \stackrel{z < 0}{=} E_i(v_1, t - z) + \frac{n_1 - n_2}{n_1 + n_2} E_i(v_1, t + z)$$

↑  
right-moving  
incident wave

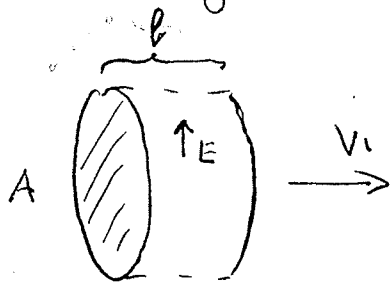
↑  
left-moving  
reflected wave

$$E(z, t) \stackrel{z > 0}{=} \frac{2n_1}{n_1 + n_2} E_i(v_1, t - \frac{v_1}{v_2} z) \leftarrow \text{right-moving (with } v = v_2 \text{)} \\ \text{transmitted wave}$$

Scattering :

before  
( $t = -\infty$ )

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$t \rightarrow -\infty$

$$E_i(v_1, t+z) = \phi \Rightarrow$$

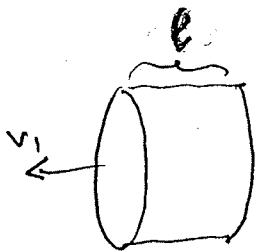
$\Rightarrow$  only incident pulse

Energy stored in the pulse

$$W_i = \epsilon_1 \int \vec{E}_i^2(\vec{r}) d^3x = \epsilon_1 \int E_0^2 d^3x = \epsilon_1 E_0^2 A l$$

volume  
of the pulse

after  
( $t = +\infty$ )



reflected pulse

( $t \rightarrow +\infty$   $E_i(z-vt)$  vanishes)

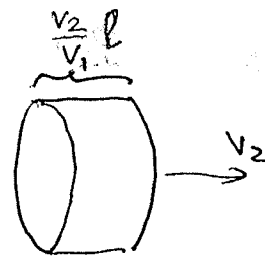
$$E_R = \frac{n_1 - n_2}{n_1 + n_2} E(v_1, t+z)$$

Energy stored in the reflected pulse

$$W_R = \epsilon_1 \int E_R^2 d^3x = \epsilon_1 \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 E_0^2 A l$$

$$\Rightarrow R = \frac{W_R}{W_i} = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$\Rightarrow R + T = 1$$



transmitted pulse

$$E_T = \frac{2n_1}{n_1 + n_2} E_i(v_1, t - \frac{v_1}{v_2} z)$$

Energy of the transmitted pulse

$$W_T = \epsilon_2 \int E_T^2 d^3x = \epsilon_2 \left( \frac{2n_1}{n_1 + n_2} \right)^2 E_0^2 A \frac{v_2}{v_1} l$$

$$W_R + W_T = W_i$$

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \left( \frac{2n_1}{n_1 + n_2} \right)^2 = \frac{n_2}{n_1} \left( \frac{2n_1}{n_1 + n_2} \right)^2$$