

Solution

(a)

$$E(r, t) = \frac{2p\hat{r}}{r^3}(1 - ikr)e^{ikr - i\omega t}$$

(b)

$$E(r, t) = \frac{2p\hat{r}}{r^3}\sqrt{1 + k^2r^2}e^{ikr - i \arctan kr - i\omega t}$$

(c)

If $E(r, t) = e^{i\Phi(r, t)}$ the phase velocity is v_p such that $\Delta\Phi = 0$ corresponds to $\Delta r = v_p\Delta t$ for small changes of r, t , and phase. For infinitesimal Δt

$$d\Phi(r, t) = \frac{\partial\Phi}{\partial r}dr + \frac{\partial\Phi}{\partial t}dt = 0 \quad \Rightarrow \quad v_p = \left. \frac{dr}{dt} \right|_{\Phi=\text{const}} = - \frac{\frac{\partial\Phi}{\partial t}}{\frac{\partial\Phi}{\partial r}}$$

In our case $\Phi(r, t) = kr - \arctan kr$ so

$$v_p = \frac{\omega}{k} \frac{1 + k^2r^2}{k^2r^2}$$

As $r \rightarrow \infty$ $v_p \rightarrow \frac{\omega}{k}$