

HW11 solution

I will use units where $c = 1$, then mass of the proton is 940 MeV, mass of the electron is ~ 0.5 MeV, and mass of the J/ψ particle is 3.1 GeV. Due to momentum conservation

$$(p_e + p_p)^2 = (k_e + k_p + k_J)^2 \quad (1)$$

Since p^2 is Lorentz invariant we can calculate it in any frame. The l.h.s in the lab frame is

$$(p_e + p_p)^2 = m_e^2 + m_p^2 + 2p_e^0 p_p^0 - 2\vec{p}_e \cdot \vec{p}_p = m_e^2 + m_p^2 + 2p_e^0 p_p^0 = m_e^2 + m_p^2 + 2E_e m_p$$

where $E_e = 6$ GeV currently and E_e will be 12 GeV for the upgraded CEBAF. As to the r.h.s. of Eq. (1), it is convenient to consider it in the c.m. frame

$$\begin{aligned} (k_e + k_p + k_J)^2 &= (E_k + E_p + E_J)^2 - (\vec{k}_e + \vec{k}_p + \vec{k}_J)^2 = (E_e + E_p + E_J)^2 \\ &= (\sqrt{k_e^2 + m_e^2} + \sqrt{k_p^2 + m_p^2} + \sqrt{k_J^2 + m_J^2})^2 \geq (m_e + m_p + m_J)^2 \end{aligned}$$

Thus, we get

$$m_e^2 + m_p^2 + 2E_e m_p \geq (m_e + m_p + m_J)^2 \simeq 16\text{GeV}^2$$

At present beam energy $m_e^2 + m_p^2 + 2E_e m_p \sim 13\text{GeV}^2$ so J/ψ particle cannot be produced while for the upgraded CEBAF $m_e^2 + m_p^2 + 2E_e m_p \sim 25\text{GeV}^2$ which permits creation of the J/ψ particle.