HW12 solution

I will use units where $c = 1$, then mass of the proton is 940 MeV, mass of the electron is ~ 0.5 MeV, and mass of the J/ψ particle is 3.1 GeV. Due to momentum conservation

$$(p_e + p_p)^2 \ = \ (k_e + k_p + k_J)^2 \tag{1}$$

Since p^2 is Lorentz invariant we can calculate it in any frame. The l.h.s in the lab frame is

$$(p_e + p_p)^2 \ = \ m_e^2 + m_p^2 + 2p_e^0 p_p^0 - 2\vec{p}_e \cdot \vec{p}_p \ = \ m_e^2 + m_p^2 + 2p_e^0 p_p^0 \ = \ m_e^2 + m_p^2 + 2E_e m_p$$

where $E_e = 6$ GeV for past CEBAF and $E_e \overset{\text{will be}}{=} 12$ GeV for the upgraded CEBAF. As to the r.h.s. of Eq. (1), it is convenient to consider it in the c.m. frame

$$(k_e + k_p + k_J)^2 \ = \ (E_k + E_p + E_J)^2 - (\vec{k}_e + \vec{k}_p + \vec{k}_J) \ = \ (E_e + E_p + E_J)^2$$
$$= \ (\sqrt{k_e^2 + m_e^2} + \sqrt{k_p^2 + m_p^2} + \sqrt{k_J^2 + m_J^2} \ \geq \ (m_e + m_p + m_J)^2$$

Thus, we get

$$m_e^2 + m_p^2 + 2E_e m_p \ \geq \ (m_e + m_p + m_J)^2 \ \simeq \ 16\text{GeV}^2$$

At the beam energy 6 GeV we have $m_e^2 + m_p^2 + 2E_e m_p \sim 13\text{GeV}^2$ so J/ψ particle cannot be produced while for the upgraded CEBAF with beam energy 12 GeV $m_e^2 + m_p^2 + 2E_e m_p \sim 25\text{GeV}^2$ which permits creation of the J/ψ particle.