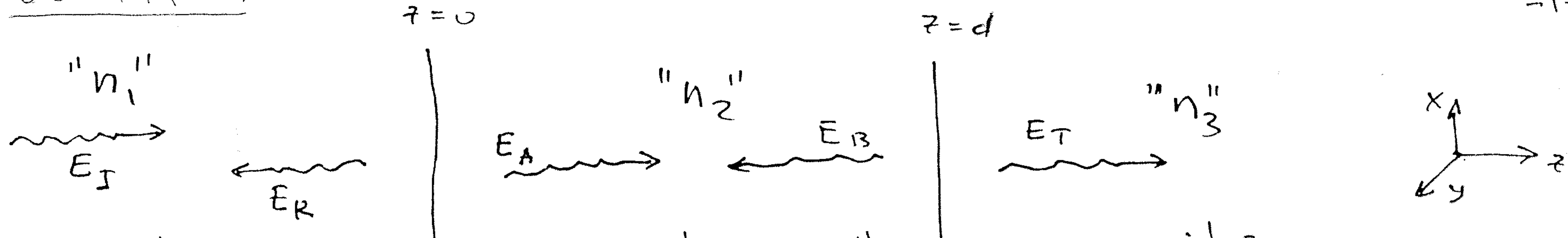
Solution



$$\vec{E} = \hat{e}_1 (E_I e^{ik_1 z} + E_R e^{-ik_1 z}) \qquad \vec{E} = \hat{e}_1 (E_A e^{ik_2 z} + E_B e^{-ik_2 z}) \qquad \vec{E} = \hat{e}_1 E_T e^{ik_3 z}$$

$$\vec{H} = \frac{\hat{e}_2}{\mu_1 v_1} (E_I e^{ik_1 z} - E_R e^{-ik_1 z}) \qquad \vec{H} = \frac{\hat{e}_2}{\mu_2 v_2} (E_A e^{ik_2 z} - E_B e^{-ik_2 z}) \qquad \vec{H} = \frac{\hat{e}_2}{\mu_3 v_3} E_T e^{ik_3 z}$$

Boundary condition at $z=0$: $E^{\parallel}$ and $H^{\parallel}$ are continuous $\Rightarrow$

$z = 0$:

$$E_I + E_R = E_A + E_B$$

$$\frac{1}{\mu_1 v_1}(E_I - E_R) = \frac{1}{\mu_2 v_2}(E_A - E_B)$$

Suppose $\mu_1 = \mu_2 = \mu_0 \Rightarrow$

$$E_I + E_R = E_A + E_B$$

$$E_I - E_R = \frac{n_2}{n_1}(E_A - E_B)$$

Boundary condition at $z = d$:

$$E_A e^{ik_2 d} + E_B e^{-ik_2 d} = E_T e^{ik_3 d}$$

$$E_A e^{ik_2 d} - E_B e^{-ik_2 d} = \frac{\mu_2 v_2}{\mu_3 v_3} E_T e^{ik_3 d}$$

$$\left.\begin{aligned} E_A e^{ik_2 d} + E_B e^{-ik_2 d} &= E_T e^{ik_3 d} \\ E_A e^{ik_2 d} - E_B e^{-ik_2 d} &= \frac{n_3}{n_2} e^{ik_3 d} E_T \end{aligned}\right\} \Rightarrow$$

$$\Leftarrow \begin{cases} 2E_A = E_T\left(1 + \frac{n_3}{n_2}\right) e^{i(k_3 - k_2)d} \\ 2E_B = E_T\left(1 - \frac{n_3}{n_2}\right) e^{i(k_3 + k_2)d} \end{cases}$$

$$4E_I = \left(1 + \frac{n_2}{n_1}\right)E_A + \left(1 - \frac{n_2}{n_1}\right)E_B =$$

$$= \left(1 + \frac{n_2}{n_1}\right)\left(1 + \frac{n_3}{n_2}\right) E_T e^{i(k_3 - k_2)d} + \left(1 - \frac{n_2}{n_1}\right)\left(1 - \frac{n_3}{n_2}\right) E_T e^{i(k_3 + k_2)d}$$

$$\Rightarrow \frac{E_T}{E_I} = \frac{4}{\left(1 + \frac{n_2}{n_1}\right)\left(1 + \frac{n_3}{n_2}\right) e^{i(k_3 - k_2)d} + \left(1 - \frac{n_2}{n_1}\right)\left(1 - \frac{n_3}{n_2}\right) e^{i(k_3 + k_2)d}}$$

$$T = \left|\frac{E_T}{E_I}\right|^2 \frac{v_1}{v_3} = \left|\frac{E_T}{E_I}\right|^2 \frac{n_3}{n_1}$$

$$\Rightarrow T = \frac{n_3}{n_1} \left| \frac{4 e^{ik_3 d}}{\left(1+\frac{n_3}{n_1}+\frac{n_2}{n_1}+\frac{n_3}{n_2}\right)e^{-ik_2 d} + \left(1+\frac{n_3}{n_1}-\frac{n_2}{n_1}-\frac{n_3}{n_2}\right)e^{ik_2 d}} \right|^2 =$$

$$= \frac{n_3}{n_1} \left| \frac{2}{\left(1+\frac{n_3}{n_1}\right)\cos k_2 d - \left(\frac{n_2}{n_1}+\frac{n_3}{n_2}\right) i \sin k_2 d} \right|^2 = \frac{4 n_1 n_3}{(n_1+n_3)^2 \cos^2 k_2 d + \frac{(n_2^2+n_1 n_3)^2}{n_2^2}\sin^2 k_2 d} \quad (*)$$

$$\Rightarrow T^{-1} = \frac{n_1}{4 n_3}\left\{\left(1+\frac{n_3}{n_1}\right)^2 \cos^2 k_2 d + \left(\frac{n_2}{n_1}+\frac{n_3}{n_2}\right)^2 \sin^2 k_2 d\right\} =$$

$$= \frac{n_1}{4 n_3}\left\{\frac{(n_1+n_3)^2}{n_1^2} + \frac{(n_2^2-n_1^2)(n_2^2-n_3^2)}{n_1^2 n_2^2}\sin^2 k_2 d\right\} = \frac{(n_1+n_3)^2}{4 n_1 n_3} + \frac{(n_1^2-n_2^2)(n_3^2-n_2^2)}{4 n_1 n_3 n_2^2}\sin^2 \frac{n_2 \omega d}{c}$$

Now R

$$2E_R = \left(1-\frac{n_2}{n_1}\right)E_A + \left(1+\frac{n_2}{n_1}\right)E_B \Rightarrow 4E_R = \left(1-\frac{n_2}{n_1}\right)\left(1+\frac{n_3}{n_2}\right)E_T\, e^{i(k_3-k_2)d} + \left(1+\frac{n_2}{n_1}\right)\left(1-\frac{n_3}{n_2}\right)E_T\, e^{i(k_3+k_2)d}$$

$$\Rightarrow \frac{E_R}{E_I} = \frac{\left(1-\frac{n_2}{n_1}\right)\left(1+\frac{n_3}{n_2}\right)e^{i(k_3-k_2)d} + \left(1+\frac{n_2}{n_1}\right)\left(1-\frac{n_3}{n_2}\right)e^{i(k_3+k_2)d}}{\left(1+\frac{n_2}{n_1}\right)\left(1+\frac{n_3}{n_2}\right)e^{i(k_3-k_2)d} + \left(1-\frac{n_2}{n_1}\right)\left(1-\frac{n_3}{n_2}\right)e^{i(k_3+k_2)d}}$$

$$\Rightarrow R = \left|\frac{E_R}{E_I}\right|^2 = \left| \frac{(1-n_3/n_1)\cos k_2 d - i\left(\frac{n_2}{n_1}-\frac{n_3}{n_2}\right)\sin k_2 d}{\left(1+\frac{n_3}{n_1}\right)\cos k_2 d - i\left(\frac{n_2}{n_1}+\frac{n_3}{n_2}\right)\sin k_2 d} \right|^2 =$$

$$= \frac{(n_1-n_3)^2\cos^2 k_2 d + \frac{(n_2^2-n_1 n_3)^2}{n_2^2}\sin^2 k_2 d}{(n_1+n_3)^2\cos^2 k_2 d + \frac{(n_2^2+n_1 n_3)^2}{n_2^2}\sin^2 k_2 d} \Rightarrow R + T = 1 \quad (\text{see } (*))$$

$$R = \frac{(n_1-n_3)^2 n_2^2 + (n_2^2-n_1^2)(n_2^2-n_3^2)\sin^2 \frac{n_2 \omega d}{c}}{(n_1+n_3)^2 n_2^2 + (n_2^2-n_1^2)(n_2^2-n_3^2)\sin^2 \frac{n_2 \omega d}{c}}$$