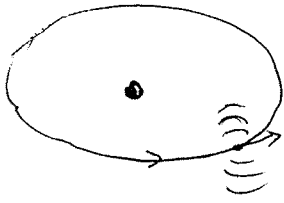


# Lifetime of the hydrogen atom in classical electrodynamics



Circular orbit

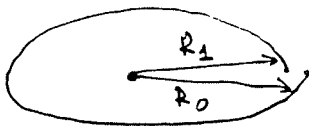
$$\frac{mv^2}{R} = \frac{e^2}{4\pi\epsilon_0 R^2} \quad \frac{v^2}{R} = \frac{e^2}{4\pi\epsilon_0 m R^2}$$

$$E_{kin} = \frac{mv^2}{2} \quad E_{pot} = -\frac{e^2}{4\pi\epsilon_0 R}$$

$$\Rightarrow E = E_{kin} + E_{pot} = -\frac{e^2}{8\pi\epsilon_0 R} \quad E_0 = -\frac{e^2}{8\pi\epsilon_0 R_0}$$

$$R_0 = 5 \cdot 10^{-11}$$

After 1 revolution



$$E_0 - E_1 \approx P T = \frac{\mu_0 e^2 a^2}{6\pi c} T \approx \frac{\mu_0 e^2 v^4}{6\pi c R_0^2} T$$

↑  
period

On the other hand

$$E_0 = -\frac{e^2}{8\pi\epsilon_0 R_0} \quad E_1 = -\frac{e^2}{8\pi\epsilon_0 R_1}$$

$$\Rightarrow E_0 - E_1 = \frac{e^2}{8\pi\epsilon_0} \left( \frac{1}{R_1} - \frac{1}{R_0} \right) \approx \frac{e^2}{8\pi\epsilon_0} \frac{R_0 - R_1}{R_0^2}$$

We get

$$\frac{R_0 - R_1}{R_0^2} \frac{e^2}{8\pi\epsilon_0} = \frac{\mu_0 e^2}{6\pi c} \frac{e^4}{16\pi^2 \epsilon_0^2 m^2 R_0^4} T \Rightarrow$$

$$\Rightarrow \frac{R_0 - R_1}{T} = \frac{\mu_0 e^4}{12\pi^2 c \epsilon_0 m^2 R_0^2} = \frac{\mu_0^2 c e^4}{12\pi^2 m^2 R_0^2} \Rightarrow$$

$$\Rightarrow \frac{\partial R(t)}{\partial t} = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2 R^2(t)} \Rightarrow R^2 dR = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2} dt$$

$$\Rightarrow \int R^2 dR = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2} \int dt \quad \frac{R^3}{3} = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2} t + \text{const}$$

$$\text{At } t=0 \quad R=R_0 \Rightarrow \text{const} = \frac{R_0^3}{3} \Rightarrow \frac{\mu_0^2 c e^4}{12\pi^2 m^2} t = \frac{R_0^3 - R^3}{3}$$

$$\Rightarrow t_{\text{lifetime}} = \frac{4\pi^2 m^2}{\mu_0^2 c e^4} R_0^3$$

$$\text{Units: } \frac{\text{kg}^2 \text{m}^3}{\frac{\text{N}^2}{\text{A}^4} \frac{\text{m}}{\text{s}} (\text{A}\cdot\text{s})^4} = \frac{\text{kg}^2 \text{m}^2}{\text{N}^2 \text{s}^3} = \text{s}$$

$$\# : t \approx 1.3 \cdot 10^{-11} \text{ s}$$