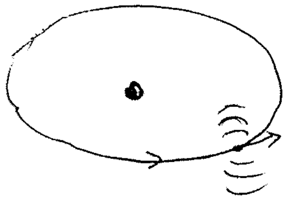
Lifetime of the hydrogen atom in classical electrodynamics



Circular orbit

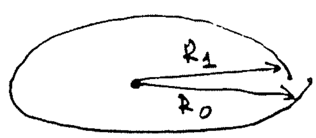
$$\frac{mv^2}{R} = \frac{e^2}{4\pi\epsilon_0 R^2} \qquad \frac{v^2}{R} = \frac{e^2}{4\pi\epsilon_0 m R^2}$$

$$E_{kin} = \frac{mv^2}{2} \qquad E_{pot} = -\frac{e^2}{4\pi\epsilon_0 R}$$

$$\Rightarrow E = E_{kin} + E_{pot} = -\frac{e^2}{8\pi\epsilon_0 R} \qquad E_0 = -\frac{e^2}{8\pi\epsilon_0 R_0}$$

$$R_0 = 5\cdot 10^{-11}$$

After 1 revolution



$$E_0 - E_1 \simeq P\,T = \frac{\mu_0 e^2 a^2}{6\pi c} T \simeq \frac{\mu_0 e^2}{6\pi c}\frac{v^4}{R_0^2} T$$

period

On the other hand $E_0 = -\dfrac{e^2}{8\pi\epsilon_0 R_0}$ $\quad E_1 = -\dfrac{e^2}{8\pi\epsilon_0 R_1}$

$$\Rightarrow E_0 - E_1 = \frac{e^2}{8\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_0}\right) \simeq \frac{e^2}{8\pi\epsilon_0}\,\frac{R_0 - R_1}{R_0^2}$$

We get

$$\frac{R_0 - R_1}{R_0^2}\,\frac{\cancel{e^2}}{8\pi\cancel{\epsilon_0}} = \frac{\mu_0 \cancel{e^2}}{6\pi c}\,\frac{e^4}{16\pi^2\epsilon_0^{\cancel{2}} m^2 R_0^4}\,T \Rightarrow$$

$$\Rightarrow \frac{R_0 - R_1}{T} = \frac{\mu_0 e^4}{12\pi^2 c\,\epsilon_0 m^2 R_0^2} = \frac{\mu_0^2 c e^4}{12\pi^2 m^2 R_0^2} \Rightarrow$$

$$\Rightarrow \frac{\partial R(t)}{\partial t} = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2 R^2(t)} \Rightarrow R^2 dR = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2} dt$$

$$\Rightarrow \int R^2 dR = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2}\int dt \qquad \frac{R^3}{3} = -\frac{\mu_0^2 c e^4}{12\pi^2 m^2} t + \text{const}$$

At $t = 0$ $R = R_0 \Rightarrow \text{const} = \dfrac{R_0^3}{3} \Rightarrow \dfrac{\mu_0^2 c e^4}{12\pi^2 m^2} t = \dfrac{R_0^3 - R^3(t)}{3}$

$$\Rightarrow t_{lifetime} = \frac{4\pi^2 m^2}{\mu_0^2 c e^4} R_0^3 \qquad \text{Units: } \frac{kg^2\, m^3}{\frac{N^2}{A^4}\,\frac{m}{s}\,(A\cdot s)^4} = \frac{kg^2 m^2}{N^2 s^3} = s$$

$\#:$ $t \simeq 1.3\cdot 10^{-11}\,s$