Phys. 807 — Statistical Mechanics

HW 1 due Tue Sept. 13 at 4 p.m. in my mailbox.

Problem 1.

A time-independent probability density for a certain physical problem has the form

$$\rho(x) \;=\; \theta(x)[\frac{a}{l}e^{-x/l} + (1-a)\delta(x-d)]$$

where a is a number between 0 and 1 and $d, l > 0$ have the dimension of length. Find mean $\bar{x}$ and rms $\sqrt{(x-\bar{x})^2}$.

Reminder: $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$.

Problem 2.

The probability density ρ for a particle moving in the x-direction with momentum p and hamiltonian $H = \frac{p^2}{2m}$ at time t_0 is given by

$$\rho(p, x, t_0) \;=\; Ce^{-\alpha(p-p_0)^2-\beta(x-x_0)^2}$$

where $\alpha, \beta > 0$. For an arbitrary time t, find $\rho(p, x, t)$, mean values $\bar{x}$, $\bar{p}$, and rms's $\sqrt{(x-\bar{x})^2}$ and $\sqrt{(p-\bar{p})^2}$.

Problem 3.

Assume that the (stationary) distribution function for the one- dimensional oscillator has the form.

$$D(x) \;=\; Ce^{-\beta\left(\frac{p^2}{2m}+\frac{m\omega^2x^2}{2}\right)}$$

where β is a positive constant. Find the normalized probability density $\rho(p, q)$ and the expectation values for p, q, p^2, q^2, and $H(p, q) = \frac{p^2}{2m} + \frac{m\omega^2x^2}{2}$.