Phys. 807 — Statistical Mechanics

HW6 (due Tue Nov 8 at 4 p.m. in my mailbox).

Consider a two-dimensional solid with a single atom per cell of size $a \times a$. Each atom is coupled to nearest neighbours only (there are 4 of them) by springs of constant $m\omega_0^2$, where m is the atom's mass. The "volume" of the solid is $L \cdot L = L^2$, and the number of atoms is N^2.

- Write down the Hamiltonian and the equations of motion for this system.
- Look for periodic solutions and determine the eigenfrequencies $\omega_\beta(\mathbf{k})$ of the matrix $\Gamma_{\alpha\beta}(\mathbf{k})$.
- Check that the operators $A_\beta(\mathbf{k})$ defined by

$$A_\beta(\mathbf{k}) = (2\hbar\omega_\beta(\mathbf{k}))^{-1/2}(\omega_\beta(\mathbf{k})Q_\beta(\mathbf{k}) + iP_\beta(-\mathbf{k}))$$

$$A_\beta^+(\mathbf{k}) = (2\hbar\omega_\beta(-\mathbf{k}))^{-1/2}(\omega_\beta(\mathbf{k})Q_\beta(-\mathbf{k}) - iP_\beta(\mathbf{k}))$$

  satisfy the commutation relations

$$[A_\beta(\mathbf{k}), A_\gamma^+(\mathbf{p})] = \delta_{\beta\gamma}\delta_{\mathbf{k},\mathbf{p}}$$

$$[A_\beta(\mathbf{k}), A_\gamma(\mathbf{p})] = [A_\beta^+(\mathbf{k}), A_\gamma^+(\mathbf{p})] = 0\,.$$

- Show that the Hamiltonian in terms of the new operators is given by the sum of Hamiltonians of quantum-mechanical oscillators corresponding to the frequences $\omega_\beta(\mathbf{k})$.
- Derive expression for the energy of this solid (similar to Debye interpolation formula) and determine the heat capacity at constant volume of this solid.