1. Consider a system of angular momentum $j = 1$, whose state space is spanned by the basis $\{ | +1 \rangle, | 0 \rangle, | -1 \rangle \}$ of three eigenvectors common to $\mathbf{J}^2$ (eigenvalue $2\hbar^2$) and J_z (respective eigenvalues $+\hbar$, 0 and $-\hbar$). The state of the system is:

$$| \psi \rangle = \alpha | +1 \rangle + \beta | 0 \rangle + \gamma | -1 \rangle$$

where α, β, γ are three given complex parameters.

a. Calculate the mean value $\langle \mathbf{J} \rangle$ of the angular momentum in terms of α, β and γ.

b. Give the expression for the three mean values $\langle J_x^2 \rangle$, $\langle J_y^2 \rangle$ and $\langle J_z^2 \rangle$ in terms of the same quantities.

2. Consider an arbitrary physical system whose four-dimensional state space is spanned by a basis of four eigenvectors $| j, m_z \rangle$ common to $\mathbf{J}^2$ and J_z ($j = 0$ or 1; $-j \leqslant m_z \leqslant +j$), of eigenvalues $j(j+1)\hbar^2$ and $m_z\hbar$, such that:

$$J_\pm | j, m_z \rangle = \hbar \sqrt{j(j+1) - m_z(m_z \pm 1)}\, | j, m_z \pm 1 \rangle$$

$$J_+ | j, j \rangle = J_- | j, -j \rangle = 0$$

a. Express in terms of the kets $| j, m_z \rangle$, the eigenstates common to $\mathbf{J}^2$ and J_x, to be denoted by $| j, m_x \rangle$.

b. Consider a system in the normalized state:

$$| \psi \rangle = \alpha | j = 1, m_z = 1 \rangle + \beta | j = 1, m_z = 0 \rangle + \gamma | j = 1, m_z = -1 \rangle + \delta | j = 0, m_z = 0 \rangle$$

(*i*) What is the probability of finding $2\hbar^2$ and $\hbar$ if $\mathbf{J}^2$ and J_x are measured simultaneously?

(*ii*) Calculate the mean value of J_z when the system is in the state $| \psi \rangle$, and the probabilities of the various possible results of a measurement bearing only on this observable.

(*iii*) Same questions for the observable $\mathbf{J}^2$ and for J_x.

(*iv*) J_z^2 is now measured; what are the possible results, their probabilities, and their mean value?