

Let \mathbf{s}_1 and \mathbf{s}_2 be the intrinsic angular momenta of two spin-1/2 particles, \mathbf{r}_1 and \mathbf{r}_2 their position observables, and m_1 and m_2 their masses. Assume the interaction V between the two particles to have the form:

$$V = v_c(r) + v_\sigma(r) \mathbf{s}_1 \cdot \mathbf{s}_2 ,$$

where $v_c(r)$ and $v_\sigma(r)$ depend only on the relative distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$ between the two particles.

1. Let $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2$ be the total spin of the two particles. Show that

$$P_1 = 3/4 + \mathbf{s}_1 \cdot \mathbf{s}_2 ,$$

$$P_0 = 1/4 - \mathbf{s}_1 \cdot \mathbf{s}_2 ,$$

are the projection operators onto the total spin states $S = 1$ and $S = 0$, respectively. Show from this that

$$V = v_0(r) P_0 + v_1(r) P_1 ,$$

where $v_0(r)$ and $v_1(r)$ are two functions of r to be expressed in terms of $v_c(r)$ and $v_\sigma(r)$.

2. Write the Hamiltonian of the “relative particle” in the center-of-mass frame, denoting with \mathbf{p} its momentum observable. Show that H commutes with \mathbf{S}^2 and does not depend on S_z . Show from this that it is possible to study separately the eigenstates of H corresponding to $S = 1$ and $S = 0$. Show that one can find eigenstates of H , with eigenvalue E , of the form

$$|\psi_E\rangle = \lambda_{00} |\psi_E^{(0)}\rangle |S = 0, M = 0\rangle + \sum_{M=-1}^{+1} \lambda_{1M} |\psi_E^{(1)}\rangle |S = 1, M\rangle ,$$

where λ_{00} and λ_{1M} are constants, $|\psi_E^{(0)}\rangle$ and $|\psi_E^{(1)}\rangle$ are kets in the \mathbf{r} -space of the relative particle. Write the eigenvalue equations satisfied by $|\psi_E^{(0)}\rangle$ and $|\psi_E^{(1)}\rangle$.