

Solution

Region 1: $x < -a$, region 2: $a > x > -a$, region 3: $x > a$.

$$\psi_1(x) = Ae^{ikx} + A_R e^{-ikx}, \quad \psi_2(x) = Ce^{ik'x} + De^{-ik'x}, \quad \psi_3(x) = A_T e^{ikx}$$

where $k = \frac{1}{\hbar} \sqrt{2mE}$, $k' = \frac{1}{\hbar} \sqrt{2m(E - V_0)}$.

Continuity of $\psi(x)$ and $\psi'(x)$:

$$\begin{array}{l} x = -a \\ Ae^{-ika} + A_R e^{ika} = Ce^{-ik'a} + De^{ik'a} \\ Ae^{-ika} - A_R e^{ika} = \frac{k'}{k}(Ce^{-ik'a} - De^{ik'a}) \\ \Rightarrow A = C \frac{k+k'}{2k} e^{i(k-k')a} + D \frac{k-k'}{2k} e^{i(k+k')a}, \quad C = \frac{k+k'}{2k'} A_T e^{i(k-k')a}, \quad D = \frac{k-k'}{2k'} A_T e^{i(k+k')a}, \end{array}$$

so

$$\begin{aligned} \frac{A_T}{A} &= e^{-2ika} \left[\frac{(k+k')^2}{4kk'} e^{-2ik'a} + \frac{(k-k')^2}{4kk'} e^{2ik'a} \right]^{-1} \\ \Rightarrow \left| \frac{A_T}{A} \right|^2 &= \left[1 + \frac{(k^2 - k'^2)^2}{4k^2 k'^2} \cos^2 2k'a \right]^{-1} \end{aligned}$$