

Problem. A beam of spineless nuclear particles of mass m and momentum p is directed along the z -axis. The particles collide with an aligned diatomic molecule but interact only with the nuclei of the molecule. If the nuclei are taken to be at $y = b$ and $y = -b$, and the constant a is positive, the interaction potential is given by

$$V(\vec{r}) = a\delta(y - b)\delta(x)\delta(z) + a\delta(y + b)\delta(x)\delta(z)$$

Calculate the scattering amplitude and the differential cross section in the Born approximation.

Solution.

In the Born approximation

$$f_k(\theta, \phi) = -\frac{m}{2\pi\hbar^2}V(\vec{k} - \vec{k}') = -\frac{m}{2\pi\hbar^2}\int d^3r' e^{i\vec{r}'\cdot(\vec{k}-\vec{k}')}V(\vec{r}')$$

and therefore

$$\begin{aligned} f_k(\theta, \phi) &= -\frac{m}{2\pi\hbar^2}\int d^3r' e^{i\vec{r}'\cdot(\vec{k}-\vec{k}')} [a\delta(y' - b)\delta(x')\delta(z') - a\delta(y' + b)\delta(x')\delta(z')] \\ &= -\frac{m}{2\pi\hbar^2}\int dx'dy'dz' e^{-ix'k'_x - iy'k'_y + iz(k-k'_z)} [a\delta(y' - b)\delta(x')\delta(z') + a\delta(y' + b)\delta(x')\delta(z')] \\ &= -\frac{ma}{2\pi\hbar^2}(e^{-ibk'_y} + e^{ibk'_y}) = i\frac{ma}{\pi\hbar^2}\cos bk'_y = \frac{ima}{\pi\hbar^2}\sin(bk \sin \theta \sin \phi) \end{aligned}$$

so the differential cross section is

$$\frac{d\sigma}{d\Omega} = |f_k(\theta, \phi)|^2 = \frac{m^2 a^2}{\pi^2 \hbar^4} \cos^2(bk \sin \theta \sin \phi)$$