Problems and solutions

Problem 1.

Consider a system of spin $\frac{1}{2}$. What are the eigenstates and eigenvalues of the operator $S_x + S_y$? Suppose a measurement of this quantity is made, and the system is found to be in the eigenstate with the larger eigenvalue. What is the probability that a subsequent measurement of S_y yields $\frac{\hbar}{2}$?

Problem 2.

Particles with angular momentum 1 are passed through a Stern-Gerlach apparatus which separates them according to the z-component of their angular momentum. Only the $m = -1$ component is allowed to pass through the apparatus. A second apparatus separates the beam according to its angular momentum component along the u-axis. The u-axis and the z-axis are both perpendicular to the beam direction but have an angle θ between them. Find the relative intensities of the three beams separated in the second apparatus.

Problem 3.

Let h_0 be the Hamiltonian of a particle. Assume that the operator $\hat{h}_0$ acts only on the orbital variables and has three equidistant levels of energies 0, $\hbar\omega_0$, and $2\hbar\omega_0$ ($\omega_0 > 0$) which are non-degenerate in the orbital state space . (In the total space, the degeneracy of each level would be $2s + 1$ where s is the spin of the particle). From the point of view of orbital variables, we are concerned only with the subspace spanned by three corresponding eigenstates of $\hat{h}_0$.

(a)

Consider a system of three independent electrons whose Hamiltonian can be written as

$$\hat{H} \;=\; \hat{h}_0(1) + \hat{h}_0(2) + \hat{h}_0(3)$$

Find the energy levels of $\hat{H}$ and their degeneracies.

(b)

Same question for a system of three identical bosons of spin 0.