Problems (from qual exams)

Problem 1.

Consider an electron (spin $\frac{1}{2}$) in a state which is an eigenstate of orbital angular momentum $l = 1$, an eigenstate of total angular momentum $j = \frac{1}{2}$, and with $j_z = \frac{1}{2}$.

Is it an eigenstate of L_z? Find an expectation value of L_z.

Problem 2.

The spin part of a wave function of a system of spin $\frac{1}{2}$ particles has the form

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \ldots \begin{pmatrix} 1 \\ 0 \end{pmatrix}_n \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{n+1} \ldots \begin{pmatrix} 0 \\ 1 \end{pmatrix}_N$$

Find the average $\hat{S}^2$ in this state (S is the total spin of these particles).

Problem 3.

A beam of spineless nuclear particles of mass m and momentum p is directed along the z-axis. The particles collide with an aligned diatomic molecule but interact only with the nuclei of the molecule. If the nuclei are taken to be at $y = b$ and $y = -b$, and the constant a is positive, the interaction potential is given by

$$V(r) \;=\; a\delta(y-b)\delta(x)\delta(z) + a\delta(y+b)\delta(x)\delta(z)$$

Calculate the scattering amplitude and the differential cross section in the Born approximation