AQM Exam.

Please put your solutions in my mailbox no later than 10 a.m. on Thu Dec 14 (or scan your solutions and e-mail to balitsky@jlab.org, cc IBalitsk@odu.edu by that time) .

Questions: (Circle the answer which you think is true.)

1. (2points). Draw a number of diagrams for the amplitude of the electron-electron scattering in QED in e^4 order of perturbation theory.
How many loops ($\equiv$ integrals $\int d^4k$) do you see in each diagram:
(a) none (b) one (c) two (d) number of loops varies from diagram to diagram

2. (2 points). A student asks a question why there is no the four-particle $ee\gamma\gamma$ vertex (like $\pi\pi\gamma\gamma$ vertex in scalar QED), and he/she proposes a new vertex $e^2 g_{\mu\nu}$ times unit matrix in spinor indices.
He/she will have problems with:
(a) relativistic invariance (b) longitudinal photons (Ward identity) (c) probabilistic interpretation (d) no problems - only on the level of experiment he/she will see that such theory gives wrong predictions.

3. (2 points). Another student proposes a $ee\gamma\gamma$ vertex of the form $e^2(k_{1\nu}k_{2\mu} - (k_1 \cdot k_2)g_{\mu\nu})$ times unit matrix in spinor indices.
She/he will have now problems with:
(a) relativistic invariance (b) longitudinal photons (Ward identity) (c) probabilistic interpretation (d) no problems - only on the level of experiment she/he will see that such theory gives wrong predictions.

Problems:

1. (3points)
Write down the momenta integrals for the matrix elements of T-matrix for the following diagrams. Solid lines denote electrons/positrons and dashed lines denote photons. (Do not calculate the integrals.)

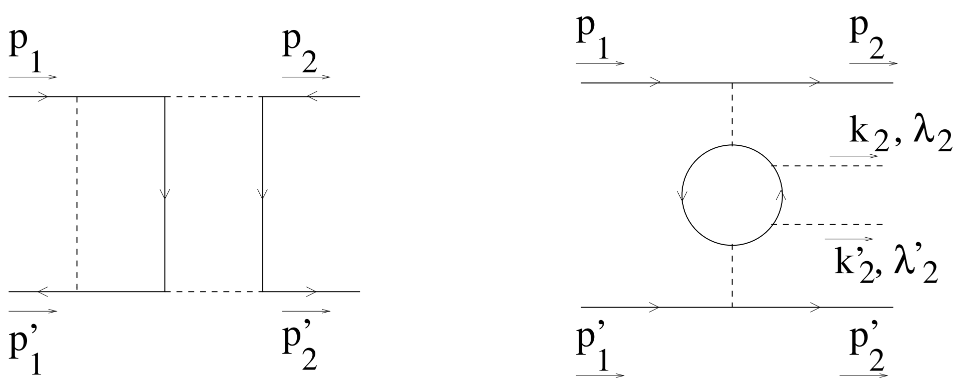

FIG. 1.
Diagrams

2. (7 points)

Find the total cross section of $\pi^+\pi^- \to \mu^+\mu^-$ annihilation in the lowest order in perturbation theory. The μ-mesons can be treated as massive electrons/positrons with mass μ. (Reminder: the notion of *total* cross section includes summation over the polarizations of final particless).

3. (3 points)

a) Write down the explicit form (= column of 4 complex numbers) of the wavefunction of an electron moving in X direction with speed v if we know that in the the rest frame the spin is pointing in Y direction.

b) Write down the explicit form (= row of 4 complex numbers) of the wavefunction of a positive-helicity positron moving in Y direction with speed v.

4. (7 points)

Find the differential cross section (in the c.m. frame) of the elastic $e^+\pi^+ \to e^+\pi^+$ scattering in the lowest order in perturbation theory. The incoming positron beam is unpolarized and the polarization of final positron is not registered.

5. (7 points)

Find the differential cross section (in the c.m. frame) of the elastic scattering of electron and μ^+ meson in the lowest order in perturbation theory. The incoming electrons and muons have positive helicity while the polarizations of final particles are not registered.

6. (7 points)

As you know, the Higgs boson (spin-0 scalar meson with mass m_H) can decay in two photons. At moderate energies, this interaction can be modeled by a local vertex, see Fig. 2a where the wavy line denote Higgs boson. Suppose this is the only vertex of interaction of Higgs bosons with photons. This vertex for the set of reduced Green functions in the momentum space has the form

$$\lambda(g^{\mu\nu} k_1 \cdot k_2 - k_2^\mu k_1^\nu)$$

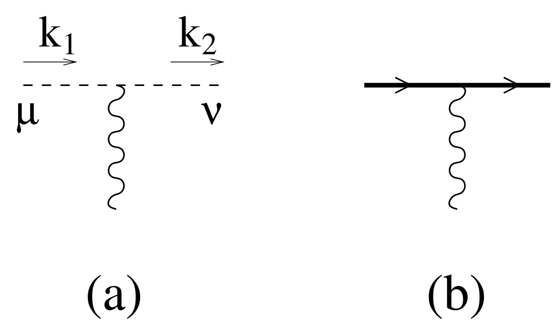

FIG. 2.
Vertices

where λ is some constant.

Suppose, in addition to the $H\gamma\gamma$ vertex shown in Fig. 2a, we have also the "eeH" vertex describing the emission of Higgs boson by the electron or positron (see Fig 2b) and let the vertex be λ ($\Leftrightarrow \frac{i\lambda}{2}$ in the coordinate space) times the unit matrix in spinor indices (so-called Yukawa vertex). Neglecting usual Compton scattering, find the total cross section of the scattering of positive-helicity photon from the unpolarized electron beam in the lowest order in λ in this model. (Recall that the notion of total cross section includes summation over the polarizations of final particles). Hint: due to Ward identity one can replace $\sum_\lambda e_\mu^\lambda(k) e_\nu^{*\lambda}(k)$ by $-g_{\mu\nu}$.

GOOD LUCK!