Solution to HW 1

1.

The three-dimensional Fourier transform of the Yukawa potential is:

$$V(\vec{q}) = \int d^3 r V_0 \frac{e^{-\alpha|\vec{r}|}}{|\vec{r}|} = \frac{4\pi V_0}{\alpha^2 + \vec{q}^2} \tag{1}$$

Substituting this expression in the general formula for the cross section for a time-independent potential (2.6.20) we get:

$$\sigma = \int \sin\theta d\theta d\phi \frac{4m^2 V_0^2}{\left[\alpha^2 + 2m^2 v_1^2(1-\cos\theta)\right]^2} \tag{2}$$

so

$$\frac{d\sigma}{d\Omega} = \frac{4m^2 V_0^2}{\left[\alpha^2 + 2m^2 v_1^2(1-\cos\theta)\right]^2} \tag{3}$$

At $\alpha = 0$ and $V_0 = \frac{Ze^2}{4\pi}$ we return to Rutherford formula (2.6.22).

2.

The total cross section is:

$$\sigma = \int \sin\theta d\theta d\phi \frac{4m^2 V_0^2}{\left[\alpha^2 + m^2 v_1^2(1-\cos\theta)\right]^2} = 8\pi m^2 V_0^2 \int_0^2 dt \frac{1}{(\alpha^2 + 2tm^2 v_1^2)^2} = \frac{16\pi m^2 V_0^2}{\alpha^2(\alpha^2 + 4m^2 v_1^2)} \tag{4}$$

3.

At $\alpha \to 0$ the total cross section goes to ∞ which means that the total cross section for Coulomb scattering is not defined. (The corresponding integral over angle θ diverges at small θ which reflects the long-distance nature of Coulomb potential).