

*Solution to HW 3*

The differential cross section for a general  $2 \Rightarrow 2$  particle cross section is calculated in Appendix A. For our case

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{"1"}} = \frac{1}{64\pi^2 s} |T|^2 \quad (1)$$

where  $s = (E_1 + E'_1)^2$  and the label "1" means that we catch the first particle (when it flies into the spherical angle  $d\Omega$ ). The relevant diagram is shown in Fig.(1). There are no diagrams of the Fig. 24b type since the particles are not identical and the diagram of the Fig. 24c type is absent because I did not specify that particle "1" can emit  $\pi$ -meson and convert into particle "2". The transition matrix is just the amputated modified Green function ,

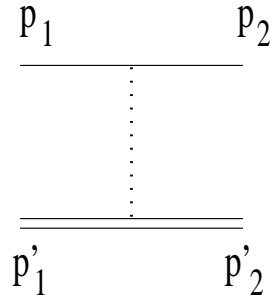


FIG. 1. Feynman diagram for the  $1, 2 \Rightarrow 1, 2$  scattering. Particle "1" is denoted by a single line and particle "2" by a double one.

$$T(p_2, p'_2; p_1, p'_1) = \frac{\lambda^2}{m^2 - t - i\epsilon} \quad (2)$$

where  $t = -4|\vec{p}_1|^2 \sin^2 \frac{\theta}{2}$ , same as before. Thus,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{"1"}} = \frac{\lambda^4}{64\pi^2 s} \left(\frac{1}{m^2 + 4|\vec{p}_1|^2 \sin^2 \frac{\theta}{2}}\right)^2 \quad (3)$$

If our detector cannot distinguish between the particles "1" and "2" you must add the corresponding cross section for the scattering into  $\pi - \theta$  angle (since in this case the "2" particle will get into our detector located at angle  $\theta$ ):

$$\left(\frac{d\sigma}{d\Omega}\right)_{1 \text{ or } 2} = \frac{\lambda^4}{64\pi^2 s} \left[ \left(\frac{1}{m^2 + 4|p_1|^2 \sin^2 \frac{\theta}{2}}\right)^2 + \left(\frac{1}{m^2 + 4|p_1|^2 \cos^2 \frac{\theta}{2}}\right)^2 \right] \quad (4)$$

Note that unlike the contributions of the Fig. 24a and b we add the cross sections rather than the amplitudes since our particles are, in principle, distinguishable and it is only because we decided to save on a cheap detector we cannot separate them.

The total cross section is just

$$\sigma_{\text{tot}} = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{„1“} = \frac{\lambda^4}{32\pi s} \int_0^\pi d\theta \sin \theta \left(\frac{1}{m^2 + 2|\vec{p}_1|^2(1 - \cos \theta)}\right)^2 \quad (5)$$

Performing the integration, we get

$$\sigma_{\text{tot}} = \frac{\lambda^4}{16\pi s} \frac{1}{m^2(m^2 + 4|\vec{p}_1|^2)} \quad (6)$$